Auction-Based Dispatch Algorithms in Deregulated Power Systems

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ABSTRACT

In deregulated power systems, the classic economic dispatch has been replaced with the auction-based dispatch in which both the generating units and loads participate. Instead of constant load demand in the classic economic dispatch, the loads are variables in the new dispatch. As a result, the conventional economic dispatch algorithms no longer can be applied. In this paper we reformulate the auction-based dispatch into a general minimization problem so that the loads can be treated as variables similar to the generations. Based on this new reformulation, efficient algorithms to solve the auction-based dispatch problem are proposed. The first algorithm solves the dispatch problem in which the objective function contains only quadratic bidding functions. Since some loads might submit linear incremental bidding functions, a second algorithm is developed to handle the situation where the objective function contains both quadratic and linear incremental bidding functions. At the end, we demonstrate the efficiency of our algorithms through examples.

1. INTRODUCTION

In the vertically regulated power systems, the economic dispatch is used to decide the generation amount among various generating units. The purpose of the classic economic dispatch is to minimize the total production cost with the total generations satisfying the demand and each generating unit within its capacity limits [1-3]. In the deregulated power systems, energy is procured through either bilateral contracts in a bilateral model market or a central auction in a poolco model market. To maintain the balance between supply and demand, efficient auction-based dispatch needs to be run frequently [4]. That calls for new and effective auction-based algorithms to solve this emerging market dispatch problem.

Both the economic dispatch and auction-based dispatch are primarily aimed to achieve economic efficiency. But, unlike the classic economic dispatch problem that has fixed demand, the auction-based dispatch in the deregulated power systems has elastic demand. In other words, both generators and loads participate in the auction. Some loads need not be covered fully and are called as responsive loads [5]. Another distinct feature for the auction-based dispatch problem is that classic economic dispatch is aimed at minimizing the total generation costs, whose costs are formulated as convex and quadratic; while the auction-based dispatch usually is to maximize the social welfare, i.e., the difference between the bidding functions of the generations and the bidding functions of the loads [6]. Besides convex quadratic bidding curves, concave quadratic and linear incremental bidding curves also occur in the auction-based dispatch [4]. The auction-based dispatch problem has the following distinct mathematical properties:

- Instead of constant loads in the classic economic dispatch, loads have become variables and the bidding functions of the loads have to be considered.

- Besides quadratic terms, the objective function might include linear incremental terms whereas the objective function of the classic economic dispatch contains only quadratic terms.
- Those variables with linear incremental bidding functions do not appear in the first order differential equations of the Kuhn-Tucker conditions. As a result, the equal incremental cost approach, which is the basis of the classic economic dispatch, will not have sufficient equations to solve the variables. Thus new methods need to be developed for such situations.

This work is focused on developing algorithms for the auction-based dispatch problem. We first reformulate the auction-based dispatch as a general minimization problem so that both generations and loads can be handled in the same way, which prepares the ground for efficiently solving the dispatch problem. Then an algorithm to solve the auction-based dispatch problem with only quadratic bidding functions is proposed. To handle the cases with both quadratic and linear incremental bidding functions, a second algorithm is developed. These two algorithms can find the optimal solution to the auction-based dispatch problem efficiently by a finite number of iterations. These iterations involve only simple algebraic calculations.

The paper is organized as follows: the auction-based dispatch problem is defined in section 2 along with the reformulation of the problem. The corresponding optimality conditions, namely the necessary and sufficient conditions, are also provided in this section. Then a dispatch algorithm with only quadratic bidding functions is introduced in section 3. The algorithm that deals with both quadratic and linear incremental bidding functions is presented in section 4. Finally several numerical examples are given in section 5 to demonstrate these two algorithms.

2. THE AUCTION-BASED DISPATCH

2.1 Formulation

In the operation of a vertically integrated power system, loads are supposed to be covered fully and generations are distributed among units by an economic dispatch program. The objective of the economic dispatch is to achieve a power balance between supply and demand with the least cost.

In a deregulated power system, the procurement of the energy is dependent on the market models. There are two principal market models in current deregulated power systems, namely the bilateral model and the poolco model. In a bilateral market, the generating units and loads enter into direct negotiation to decide the power quantities and prices. In a poolco market, the amounts of generations and loads are determined by the auction-based dispatch that basically is an optimization problem that deals with the supply and demand of power. Therefore the auction-based dispatch in a power market should be conducted based on the principles of supply and demand. A typical formulation of the auction-based dispatch in a deregulated environment is shown as below [4].

Objective:

$$Max \quad f = \left(\sum_{j=1}^{l} D_j(P_{lj}) - \sum_{i=1}^{m} C_i(P_{Gi}) \right) \tag{1}$$

Subject to:

$$\sum_{i=1}^{m} P_{Gi} - \sum_{j=1}^{l} P_{lj} = 0 \tag{2}$$

$$P_{Gi\,min} \le P_{Gi} \le P_{Gi\,max} \qquad i = 1,...,m \qquad (3)$$

$$P_{ij\,min} \le P_{ij} \le P_{ij\,max} \qquad j = 1,...,l \qquad (4)$$

where, P_{ij} = Real power amount of j th load,
$P_{ij\,max}, P_{ij\,min}$ = Maximum and minimum requirements of j th load,
P_{Gi} = Real power amount of j th generator,
$P_{Gi\,max}, P_{Gi\,min}$ = Real power limits of j th generator,
i = Number of loads,
m = Number of generators,
$D_j(P_{Lj})$ = Bidding function of j th load, and
$C_i(P_{Gi})$ = Bidding function of generator.

The objective function is the bidding cost difference between the loads and generators. In other words, the auction-based dispatch is to maximize the social welfare.

Generally, a quadratic function with positive coefficients is used to approximate the cost function of a generator [4, 6]. Therefore the bidding functions of generators have the following form:

$$C_i(P_{Gi}) = d_{Gi}P_{Gi}^2 + e_{Gi}P_{Gi} + f_{Gi} \quad (d_{Gi} > 0, e_{Gi} > 0, f_{Gi} > 0) \qquad (5)$$

We can see that these bidding functions are convex.

Like generators, a quadratic function is adopted to represent a load's bid. From the theory of economics [7], we know that the differentials of the bidding curves of generators and loads are essentially supply and demand curves respectively. Thus the demand curve of the load is a linear function. In terms of the law of demand, we can know that usually the power quantity demanded is inversely related to the price, i.e., marginal cost. That means the demand curve has a negative slope. Some loads might have a perfectly elastic demand, i.e., a zero slope for the demand curve. Therefore the general form for the bidding functions of loads can be given as follows:

$$D_j(P_{ij}) = d_{ij}P_{ij}^2 + e_{ij}P_{ij} + f_{ij} \quad (d_{ij} \le 0, e_{ij} > 0) \qquad (6)$$

In the above equation, $d_{ij} = 0$ means the corresponding load has a perfectly elastic demand. Eq. (6) implies besides concave quadratic bidding functions, loads can also submit linear incremental bidding functions.

It can be proved that with this formulation, the dispatch is run at the equilibrium point of the market supply and demand curves.

From the above formulation, we can see that the auction-based dispatch is more complicated than the classic economic dispatch in the following aspects:

- Loads no longer need to be covered completely.
- Since loads are variables now, the total amount of generations becomes uncertain.
- The objective function is not a simple summation of cost functions of generators. It becomes the difference of bidding costs between the loads and generations. In addition to quadratic bidding curves, linear incremental bidding curves are also involved in the objective function.

2.2 Reformulation of the Problem

In terms of the properties of the bidding functions of generators and loads, we can modify our formulation so that the auction-based dispatch can be solved efficiently.

First, we modify the objective function.

$$(1) \Rightarrow Min \quad F = -f = \sum_{i=1}^{m} C_i(P_{Gi}) - \sum_{j=1}^{l} D_j(P_{ij})$$
$$= \sum_{i=1}^{m} C_i(P_{Gi}) + \sum_{j=1}^{l} \left(-D_j(P_{ij})\right) \qquad (7)$$

Then let $P_{ij}^{New} = -P_{ij}$.

With these two changes, we can reformulate the auction-based dispatch problem as a general minimum optimal problem:

Objective:

$$Min \quad F = \sum_{i=1}^{n} B_i(x_i) = \sum_{i=1}^{n} (a_i x_i^2 + b_i x_i + c_i) \qquad (8)$$

Subject to:

$$\sum_{i=1}^{n} x_i = 0 \qquad (9)$$

$$x_{i\min} \le x_i \le x_{i\max} \qquad i = 1,\ldots,n \qquad (10)$$

where, x_i = P_{Gi} or P_{ij}^{New} ,
B_i = represents the bidding function of or , and
n = the number of and is equal to , the total number of generators and loads.

After the reformulation, a term of the objective function, is a convex quadratic function with $a_i > 0$ or a linear incremental function with $a_i = 0$. $x_i \ge 0$ if this variable represents a generator while $x_i \le 0$ if it corresponds to a load.

2.3 Optimality Conditions

The necessary conditions for the problem (8 - 10) can be easily got based on the Kuhn-Tucker conditions [8 - 10]:

$$\begin{cases} \dfrac{dB_i}{dx_i} = \lambda & for \quad x_{i\,min} < x_i < x_{i\,max} \\ \dfrac{dB_i}{dx_i} \le \lambda & for \quad x_i = x_{i\,max} \\ \dfrac{dB_i}{dx_i} \ge \lambda & for \quad x_i = x_{i\,min} \end{cases} \qquad (11)$$

where, λ = the Lagrange multiplier.

The above conditions are also the sufficient conditions as the objective function is convex and the constraints are linear.

3. AN ALGORITHM TO SOLVE THE AUCTION-BASED DISPATCH WITH QUADRATIC BIDDING FUNCTIONS

In this section we will develop an algorithm to solve the auction-based dispatch problem (1-4) by assuming all bidding functions are quadratic functions. For the case that includes both quadratic and linear incremental curves, the solution can be found in the section 4. To make the problem (1-4) meaningful, we assume $\sum_{i=1}^{m} P_{Gi\min} \le \sum_{j=1}^{l} P_{Lj\max}$ and $\sum_{j=1}^{l} P_{Lj\min} \le \sum_{i=1}^{m} P_{Gi\max}$. Otherwise there would be no solution.

Algorithm

1) Rewrite the auction problem (1 - 4) into an optimization problem (8 - 10).
2) Define $M = \varnothing$ and $t = 0$.
3) Get λ and all variables $x_i (i \notin M)$ according to (12, 13).

$$\lambda = \frac{t + \sum_{i=1}^{nk} \frac{b_i}{2a_i}}{\sum_{i=1}^{nk} \frac{1}{2a_i}} \tag{12}$$

$$x_i = \frac{\lambda - b_i}{2a_i} \quad i = 1,..., nk \tag{13}$$

(where $a_i > 0 (i = 1,..., nk)$ and nk is the number of variables $x_i (i \notin M)$.)

4) If all $x_i (i \notin M)$ are within the limits, go to 7). Otherwise, if $x_i > x_{i\max}$, set x_i to $x_{i\max}$; If $x_i < x_{i\min}$, set x_i to $x_{i\min}$.
5) Let $S = \sum_{i \notin M} x_i$
   - If $S = t$ or $|S - t| \le \varepsilon$, (ε is a specified small number), go to 7).
   - If $S > t$, let $L = \{i | x_i = x_{i\min}, i \notin M\}$ and $M = M \cup L$.
   - If $S < t$, let $U = \{i | x_i = x_{i\max}, i \notin M\}$ and $M = M \cup U$.
6) $t = -\sum_{i \in M} x_i$. Go back to 3).
7) Convert x back into P_G, P_L. Print the result and stop.

Eqs. (12, 13) are obtained by using the necessary conditions (11) with the assumption that all variables $x_i (i \notin M)$ are within the limits.

In step 4), if all the variables $x_i (i \notin M)$ obtained by (13) do not violate the limits, which means they satisfy the necessary conditions (11) and the solution is optimal. If some variables violate the limits, we fix these variables at their violated upper or lower limits. Based on the knowledge that $\frac{dB_i}{dx_i} = 2a_i x_i + b_i \quad (a_i > 0)$ for every variable x_i, we know that these variables with the new values satisfy the necessary conditions. If the equality constraint, i.e., $S = \sum_{i \notin M} x_i = t$, is also satisfied, then we can claim the solution is optimal. If the equality constraint does not hold, we will make an adjustment in step 5) by comparing S and t. Suppose $S > t$. Then we know that

- The current λ is bigger than $\lambda_{optimal}$, the optimal solution. We can prove this by contradiction. Suppose $\lambda_{optimal}$ is bigger than λ. For the optimal solution, $S_{optimal} = \sum_{i \notin M} x_{ioptimal} = t$. Since $a_i > 0 (i \notin M)$ in (13), the assumption $\lambda_{optimal} > \lambda$ implies that $x_{ioptimal} \geq x_i (i \notin M)$ (They might be equal if x_i is at the limit and (13) can not be applied). Therefore we can get $S_{optimal} = \sum_{i \notin M} x_{ioptimal} \geq \sum_{i \notin M} x_i = S$. On the other hand $S > t$. That implies $S_{optimal}$ is larger than *t* instead of equal to *t*, which means our assumption is wrong and $\lambda_{optimal}$ can only be smaller than λ.
- Based on the conclusion that the current λ is too big and (13), those variables at the lower limits in this iteration must also be at the lower limits for the final optimal solution. In other words, the variables at the set $L = \{i | x_i = x_{i\min}, i \notin M\}$ will remain at the lower limits in the future steps.
- The set L is not empty. From step 3), the sum of the variables not belonging to set M is equal to t. Step 4) makes an adjustment, by decreasing those violating the upper limits to their upper limits and increasing those violating the lower limits to their lower limits. If L is empty, that implies no variable is increased. As a result, the sum of the variables after the adjustment, namely S, is not larger than the sum before the adjustment, namely t. That contradicts the condition $S > t$. Therefore the set L cannot be empty and we fix at least one more variable to the lower limits when $S > t$.
- Accordingly, fixing the lower limits will increase the sum of the corresponding variables. To maintain the same total amount t, the sum of the other variables not in set M has to decrease. Then by (13), λ_{new} in the next iteration must decrease; and thus λ_{new} is smaller than $\lambda_{current}$, the current λ. Those variables fixed at the lower limits have $\frac{dB_i}{dx_i} \geq \lambda_{current}$, we can conclude $\frac{dB_i}{dx_i} > \lambda_{new}$. As a consequence, the necessary conditions (11) are still satisfied for these variables. Accordingly, these variables remain fixed at the lower limits in the new iteration.

- Some more conclusions can be drawn based on the same logic. If the first iteration only violates lower limits, we will have $S > t$ after fixing the lower limits. The above analysis tells us when the lower limits are fixed, the remaining variables have to decrease and the Lagrange multiplier λ will decrease. That implies no new upper limit violations will be created. In the meantime, λ will keep decreasing until the optimal one. Similar arguments can apply to the case that only upper limits are violated. However, if the first iteration violates both lower and upper limits and $S > t$, then we might still need to fix the upper limits after fixing the lower limits. For the case that both upper and lower limits are violated and suppose that it begins with $S > t$, we repeat fixing the lower limits until iteration h, which has $\lambda_h < \lambda_{optimal}$ or $S_h < t_h$. Then we should fix the upper limits. After that, $\lambda_{(h+1)}$, the Lagrange multiplier in the following iteration, is greater than λ_h. But $\lambda_{(h+1)}$ will not be bigger than $\lambda_{(h-1)}$, the Lagrange multiplier in iteration $(h-1)$. That is $\lambda_h < \lambda_{(h+1)} < \lambda_{(h-1)}$. These conclusions can be proved by contradiction. We can further prove that even though the process of fixing the upper limits may involve more than one iteration and the λ keeps increasing, the biggest Lagrange multiplier will not be larger than $\lambda_{(h-1)}$. After fixing the upper limits, we might need to fix the lower limits again. Again, it is easy to prove that the new λ will locate between the previous two values, one of which is bigger than $\lambda_{optimal}$, the other is less. In other words, λ oscillates around but gets closer to the optimal one. Eventually it settles at the optimal value.
- Similar conclusions can apply to $S < t$. The above analysis implies that our algorithm, by fixing the lower or upper limits, will make the Lagrange multiplier λ approach and finally settle at the optimal one. And the variables that are fixed at the lower or upper limits will remain fixed during the future iterations.

As a summary, before the optimal solution is obtained, during each iteration from steps 3) to 6), at least one more variable will be fixed at either the upper limit or the lower limit; and the fixed ones will remain fixed during future iterations. As there are only n variables, the optimal solution will be obtained with at most iterations. The iteration only involves simple algebraic calculations (See (12, 13)) and the number of variables participating in (12, 13) is decreasing. Therefore this algorithm is very efficient.

4. AN ALGORITHM TO SOLVE THE AUCTION-BASED DISPATCH WITH BOTH QUADRATIC AND LINEAR INCREMENTAL BIDDING FUNCTIONS

Besides quadratic bidding curves, loads can have linear incremental bidding curves with $D_j(P_{ij}) = d_{ij}P_{ij}^2 + e_{ij}P_{ij} + f_{ij} \quad (d_{ij} = 0, e_{ij} > 0)$. Here we assume that these e_{ij} are different. (If some e_{ij} are identical, we can combine the corresponding variables together.) By the necessary conditions (11), we know that variables with linear incremental bidding curves will have at most one variable within the limits; and others either at the upper or the lower limits.

We define Z as the set whose variables only have linear incremental bidding curves and set Y that contains variables with quadratic bidding curves. Hence we can solve λ in (12) by the following equation.

$$\lambda = \lim_{a \to 0} \left(\frac{t + \sum_{i \in Y} \frac{b_i}{2a_i} + \sum_{i \in Z} \frac{b_i}{2a}}{\sum_{i \in Y} \frac{1}{2a_i} + \sum_{i \in Z} \frac{1}{2a}} \right)$$

$$= \lim_{a \to 0} \left(\frac{2a\left(t + \sum_{i \in Y} \frac{b_i}{2a_i} \right) + \sum_{i \in Z} b_i}{a \sum_{i \in Y} \frac{1}{a_i} + z} \right)$$

$$= \frac{\sum_{i \in Z} b_i}{z} \tag{14}$$

In the above equation, z refers to the number of variables in set Z and y is the number of variables in set Y.

Based on the above information, we develop an algorithm for the auction-based dispatch problem including both quadratic and linear incremental bidding curves.

1) Rewrite the auction problem (1 - 4) into an optimization problem (8 - 10).
2) Define $M = \varnothing$ and $t = 0$.
3) Get λ according to (14) for all variables in sets Z, Y.
4) For the variables in Z, if $\lambda > b_i$, set the corresponding x_i to $x_{i\max}$; if $\lambda < b_i$, set the corresponding to x_i to $x_{i\min}$.
5) For the variables in Y, calculate x_i in terms of the following equation

$$x_i = \frac{\lambda - b_i}{2a_i} \quad (i \in Y) \tag{15}$$

If $x_i > x_{i\max}$, set the corresponding x_i to $x_{i\max}$; If $x_i < x_{i\min}$,set the corresponding x_i to $x_{i\max}$. Otherwise, x_i remains the same value, i.e., still within the limits.

6) If $\lambda \neq b_i$ for all $i \in Z$, then let $S = \sum_{i \in Z} x_i + \sum_{i \in Y} x_i$.
   - If $S = t$ or $|S - t| \leq \varepsilon$, (ε is a specified small number), go to 10).
   - If $S > t$, let $L = \{i | \lambda - b_i < 0, i \in Z\}$ and $Z = Z - L$.
   - If $S < t$, let $U = \{i | \lambda - b_i > 0, i \in Z\}$ and $Z = Z - U$.

If $\lambda = b_k \, (k \in Z)$, then $S = \sum_{i \in Z, i \neq k} x_i + \sum_{i \in Y} x_i$

- If $S + x_{k\,\min} \le t \le S + x_{k\,\max}$, let $x_k = t - S$. Go to 10).
- If $S + x_{k\,\min} > t$, Let $L = \{ i | \lambda - b_i \le 0, i \in Z \}$ and $Z = Z - L$.
- If $S + x_{k\,\max} < t$, let $U = \{ i | \lambda - b_i \ge 0, i \in Z \}$ and $Z = Z - U$.

7) If $S > t$ (or $S + x_{k\,\min} > t$), set the variables in L to the lower limits and let $M = M \cup L$. Otherwise, set the variables in U to the upper limits and $M = M \cup U$.

8) $t = -\sum_{i \in M} x_i$.

9) If $Z \neq \varnothing$, go back to 3). Otherwise, employ "the algorithm for the auction-based dispatch with quadratic bidding functions" to get the solution for variables in set Y.

10) Convert x back into P_G, P_L. Print the result and stop.

This algorithm smartly seeks the solution that satisfies the optimality conditions.

Notice that in the above algorithm set Z is changing during the iterations. Here we use z_0 to represent the initial number of variables in Z. Suppose initially $b_1 < b_2 < \cdots < b_{z_0}$ for set Z. Evidently $\lambda_{optimal}$, the λ of the necessary conditions (11) can only be one of the 3 possibilities, i.e., $\lambda_{optimal} > b_{z_0}$, $b_{z_0} \ge \lambda_{optimal} \ge b_1$ or $\lambda_{optimal} < b_1$.

We first get λ by (14). Obviously, $b_{z_0} \ge \lambda \ge b_1$. Assume $\lambda \neq b_i$ for all $i \in Z$. (If $\lambda = b_k \, (k \in Z)$, we can handle it in a similar way.) In step 6), if $S = t$, the results are the optimal solution as the necessary conditions are satisfied. Otherwise, $S \neq t$, which means $\lambda \neq \lambda_{optimal}$. Then we need to decide whether to increase λ or decrease λ. If $S > t$, increasing λ might lead to some variables in Z changing from the lower limits to the upper limits with other variables unchanged. Meanwhile all variables in Y will increase by using (15). After we fix the upper and lower limits for set Y, the sum of set Y is still bigger than the previous value. As a result, S will become larger if we increase λ. That implies we need a smaller λ. To decrease λ, we let $Z = Z - L$, i.e., we remove variables with $b_i > \lambda$. Hence the new λ by Eq. (14) will become smaller. If $S < t$, we can deal with Z by removing variables with $b_i < \lambda$.

Initially the number of variables in Z is z_0. Before the optimal solution is obtained, during each iteration from steps 3) to 8), at least one more variable in Z will be fixed at the limit and removed from Z. Then based on (14), we can find out whether the $\lambda_{optimal}$ is within or beyond $b_i \, (i = 1,\ldots, z_0)$ with at most z_0 iterations. And each iteration just involves simple algebraic calculations.

If the $\lambda_{optimal}$ is located between $b_i \, (i = 1,\ldots, z_0)$, i.e., $b_{z_0} \ge \lambda_{optimal} \ge b_1$, steps 1) ~ 8) can solve the problem with at most z_0 iterations. If $\lambda_{optimal}$ is beyond $b_i \, (i = 1,\ldots, z_0)$, we need to resort to the algorithm developed in section 3 to get the solution for the variables in set Y, namely variables with quadratic bidding functions. From the analysis in the preceding section, we can know that it will take at most y iterations to find out the optimal solution for them. Therefore, totally it will take at most $(z_0 + y)$ iterations for our algorithm to get the optimal solution of the auction-based dispatch with both quadratic and linear incremental bidding functions.

5. NUMERICAL EXAMPLES

The proposed algorithms will be tested on some numerical examples. We will first consider cases with quadratic bidding functions only. Then we will include linear incremental bidding functions in the auction-based dispatch.

5.1 Quadratic Bidding Functions Only

Case 1:

Table 1 Data of generators and loads

	Type	Bidding data ($/h)	Limits (MW)
1	Generator	$0.01P_{G1}^2 + 12P_{G1} + 300$	[50, 500]
2	Generator	$0.012P_{G2}^2 + 6P_{G2} + 400$	[100, 500]
3	Load	$-0.016P_{L1}^2 + 35P_{L1}$	[0, 400]
4	Load	$-0.017P_{L2}^2 + 34P_{L2}$	[0, 700]

By applying the algorithm for the auction-based dispatch with quadratic bidding functions, we first get the solution of the optimization problem (8 - 10) as:

$$x = [344.44 \quad 500 \quad -400 \quad -444.44]$$
$$\lambda = [19.3250 \quad 18.5830 \quad 18.8889]$$

It is noted that:

- The negative signs are for load variables.
- The optimization problem (8 - 10) is solved by 3 iterations. Thus the λ has three values. And the value of oscillates and converges to the optimal value.
- The solution satisfies the optimality conditions.

And similar conclusions can be made for the following cases.

After converting the above solution back to generations and loads, we can obtain the solution to the auction problem (1 - 4) as follows:

$$P_G = [344.44 \quad 500](MW)$$
$$P_L = [400 \quad 444.44](MW)$$

It can be seen that generator 2 and load 1 hit the upper limits while others are within the limits.

Case 2:

Table 2 Data of generators and loads

	Type	Bidding data ($/h)	Limits (MW)
1	Generator	$0.01P_{G1}^2 + 12P_{G1} + 300$	[50, 200]
2	Generator	$0.011P_{G2}^2 + 13P_{G2} + 400$	[100, 310]
3	Generator	$0.009P_{G3}^2 + 11P_{G3} + 300$	[50, 400]
4	Generator	$0.0095P_{G4}^2 + 14P_{G4} + 400$	[100, 300]
5	Generator	$0.05P_{G5}^2 + 15P_{G5} + 300$	[50, 200]
6	Generator	$0.013P_{G6}^2 + 6P_{G6} + 400$	[100, 550]
7	Load	$-0.03P_{L1}^2 + 10P_{L1}$	[0, 300]
8	Load	$-0.017P_{L2}^2 + 35P_{L2}$	[0, 500]
9	Load	$-0.015P_{L3}^2 + 35P_{L3}$	[0, 600]
10	Load	$-0.018P_{L4}^2 + 33P_{L4}$	[0, 700]
11	Load	$-0.0182P_{L5}^2 + 37P_{L5}$	[0, 400]

The solution of the problem (8 - 10) is:

$$x = \begin{bmatrix} 200 & 294.1314 & 400 & 287.9416 & 50 & 518.1112 \\ 0 & -456.7385 & -517.6370 & -375.8086 & -400 & \end{bmatrix}$$
$$\lambda = [18.6561 \quad 19.7992 \quad 19.4709]$$

The above results show that though there are more variables than case 1, case 2 takes the same number of iterations to find the optimal solution.

Based on the above solution to the optimization problem (8 - 10), we can easily get the solution of the auction problem (1 - 4):

$$P_G = [200 \quad 294.1314 \quad 400 \quad 287.9416 \quad 50 \quad 518.1112](MW)$$
$$P_L = [0 \quad 456.7385 \quad 517.6370 \quad 375.8086 \quad 400](MW)$$

Generators 1 and 3 hit the upper limits while generator 5 hits the lower limit. Load 5 is at the upper limit and load 1 is at the lower limit.

5.2 With Quadratic and Linear Incremental Bidding Functions

Case 1:

Table 3 Data of generators and loads

	Type	Bidding data ($/h)	Limits (MW)
1	Generator	$0.01P_{G1}^2 + 12P_{G1} + 300$	[0, 400]
2	Generator	$0.015P_{G2}^2 + 6P_{G2} + 400$	[0, 300]
3	Load	$34P_{L1}$	[0, 300]
4	Load	$35P_{L2}$	[0, 500]

By implementing the algorithm for the auction-based dispatch with both quadratic and linear incremental bidding functions, we first get the solution of the problem (8 - 10):

$$x = [400 \quad 300 \quad -200 \quad -500]$$
$$\lambda = [34.5 \quad 34]$$

In this case, only 2 iterations are needed for finding the optimal solution. Hence, the solution to the auction problem (1 - 4) is:

$$P_G = [400 \quad 300](MW)$$
$$P_L = [200 \quad 500](MW)$$

And it can be seen that load 2 is at the upper limit whereas load 1 is within the limits. This is expected as the bidding cost of load 2 is always higher than that of load 1, load 1 cannot get any power unless load 2 is fully supplied.

Case 2:

Table 4 Data of generators and loads

	Type	Bidding data ($/h)	Limits (MW)
1	Generator	$0.01P_{G1}^2 + 12P_{G1} + 300$	[0, 400]
2	Generator	$0.015P_{G2}^2 + 6P_{G2} + 400$	[0, 300]
3	Generator	$0.011P_{G3}^2 + 11P_{G3} + 300$	[0, 400]
4	Generator	$0.013P_{G4}^2 + 13P_{G4} + 400$	[0, 300]
5	Load	$34P_{L1}$	[0, 300]
6	Load	$35P_{L2}$	[0, 200]
7	Load	$33P_{L3}$	[0, 300]
8	Load	$36P_{L4}$	[0, 200]
9	Load	$37P_{L5}$	[0, 100]
10	Load	$-0.03P_{L6}^2 + 25P_{L6}$	[0, 150]

Again, by using the algorithm for the auction-based dispatch with both quadratic and linear incremental bidding functions, we can get the solution of the problem (8 - 10):

$$x = \begin{bmatrix} 335.2554 & 300 & 350.2322 & 219.4272 & & \\ -300 & -200 & -300 & -200 & -100 & -104.9149 \end{bmatrix}$$
$$\lambda = [35 \quad 33.5 \quad 33 \quad 18.0336 \quad 18.7051]$$

From the values of the λ , we know that actually the algorithm in section 3, i.e., the one for the auction-based dispatch with quadratic bidding functions, is also involved in the process. It first takes 3 iterations to judge the location of $\lambda_{optimal}$. After finding out that $\lambda_{optimal}$ is smaller than the marginal costs of the variables with linear incremental bidding curves, we employ the algorithm in section 3 with 2 more iterations to find the solution. Therefore, in total, it takes 5 iterations to obtain the optimal solution.

The solution of the auction problem (1 - 4) is:

$$P_G = [335.2554 \quad 300 \quad 350.2322 \quad 219.4272](MW)$$
$$P_L = [300 \quad 200 \quad 300 \quad 200 \quad 100 \quad 104.9149](MW)$$

The above answer indicates that loads 1 to 5 hit the upper limits while load 6 is within its limits. The reason is that the bidding costs of loads 1 to 5 are so high that they should be covered first before load 6 can be supplied.

6. CONCLUSIONS

This paper develops the auction-based dispatch algorithms in the deregulated power systems. With appropriate formulations, two algorithms solving the auction-based dispatch with quadratic only, and mixed quadratic and linear incremental bidding functions are presented. Both algorithms will have iteration complexity less than the order of the number of variables. Each iteration requires very basic algebraic calculations. Hence the two algorithms are very effective and efficient. The numerical examples demonstrate the efficiency and accuracy of the proposed approaches.

7. ACKNOWLEDGEMENTS

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