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Equilibrium Prices in Transmission Constrained Electricity Markets: Non-Linear Pricing and Congestion Rents

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ABSTRACT

The use of transmission constrained unit commitment models to determine nodal energy prices in electricity markets is widely recognized as an efficient way implement electricity market that provide efficient pricing signals in the short term. However, complexities of unit commitment models arising from non-convexities in generation cost functions and network models can cause the absence of classical Walrasian equilibrium; this means that the unit commitment based dispatch model will not be able, under some situations, to find a single equilibrium price for which the energy market is cleared. Based on recent research on non-linear pricing for single-node unit commitment models, this paper proposes a transmission constrained non-linear pricing alternative based coordination functions added to the classic decomposition Lagrangian relaxation algorithm to solve transmission constrained unit commitment models. The new coordination algorithms finds agent's purchase and sell prices that coordinate the market in the obscene of classic equilibrium.

Since non-linear prices differ for each agent connected to a transmission node, the value of congestion rents is redefined so that the new pricing mechanism is taken in to account. The redefinition of congestion rents is necessary so that the implementation of financial transmission rights, in their varied forms, is still possible in order to provide a price hedging mechanism in the nodal spot market.

1. INTRODUCTION

The evolution of the electricity industry, from a centralized and regulated situation, towards a new philosophy of competition, has generated a wide range of organizational structures and market designs [1-2]. Regarding spot market design there is a need to build a model that finds prices that clear the market, in the Walrassian sense [3], for all the conditions in the transmission constrained system [4, 5], and do not contain design elements that can prevent workable competition to be realized at the spot market level. In this paper, we dealt with power "Pool" energy auctions where central unit commitment is used as the market clearing mechanisms such as in the original England and Wales Pool [6], and the current PJM and NY-ISO markets [1].

Contrary to the goods or articles that are commercialized in other markets, electric power cannot be stored; balance among power supply and demand has to take place at every time instant in the system; in addition, several non-convex technical and security constraints have to be observed at every time in the system [4]. For these reasons, the electricity market's clearing mechanisms have to take into account such characteristics, specially the transmission system and generators capabilities. The incorporation of such characteristics in unit commitment models for the spot market require extra considerations for its solution and, specially, for price determination.

For the solution to Transmission Constrained Unit Commitment (TCUC) models, the Lagrangian Relaxation (LR) technique has become an industry standard due to its reliability to solve for large systems. Most of LR variations to solve TCUC models consist of the following stages [9, 10, 12-14, 24]: i) adding duplication variables in the primal problem to handle transmission constraints, ii) adding artificial penalty-type constraints to reduce oscillations in the solution to the dual of the new Augmented Lagrangian problem; iii) applying the Auxiliary Problem Principle (APP) to enable separation of the new penalty terms in the Augmented Lagrangian; iv) solving the dual problem to a particular optimality criteria using one of several existing techniques for non-differentiable optimization, v) using heuristics, if needed, to find a primal feasible solution.

Price determination for unit commitment power pool auctions became an imperative design issue in different electricity markets, non-convexities in the unit commitment models that difficult pricing [16] and difficultness associated with parameter tuning [28] of pioneer LR techniques have been overcome over the time. In [16] it was recognized that such non-convexities in UC models prevent, under particular situations, the existence of classic equilibrium, and create conflicts of interests since the scheduled agents may be forced to operate to a non-profit maximizing state. In [7, 8] the first non-linear, or extended pricing approach, to cope with the pricing problem of single-node UC models was proposed along with new non-differentiable optimization techniques to cope with parameter tuning issues in the LR algorithm. In [17] a linear programming approximation to non-convex integer, unit commitment-like problems, is proposed as an alternative to find clearing prices, the proposed approach is applied also to small-scale uni-nodal unit commitment models. In [18], a non-linear pricing algorithm based on agents coordination trough disincentive functions is proposed, the method is able to coordinate agents and eliminate the duality gap when equilibrium does not exists in the energy auction. In [19] and [20] formalizations of price uplifts, that have been used in other ad-hoc versions in markets such as PJM [27], are proposed to cope with the same problem. All the non-linear pricing methodologies so far proposed have been able to overcome the initial difficultness but only for pricing single-node unit commitment models. However, recognizing that nodal pricing is considered necessary in different markets designs to provide efficient locational and congestion price signals [1], there is still a need to identify and solve the pricing problem when transmission constrained unit commitment models are used for the spot market.

This paper shows that under similar circumstances to the single-node case, dual variables from transmission-constrained LR unit commitment models, nodal prices do not represent market clearing prices and we propose a non-linear pricing scheme for such situation when; the objective of the scheme is to find prices that clear the market when “marginal costs” from dual variables out of the LR are not in equilibrium and force agents to operate at a loss. Our approach is based on a simplification the approach in [18] applied to single-node UC models. Since the proposed Non-Linear Coordinated Simplified Pricing (NLCSP) for TCUC electricity auctions leads to different prices for each agent connected to a transmission node, the paper also proposes a new definition of congestion rents which is now required in the absence of unique nodal prices. Financial Transmission Rights (FTR), in their varied forms [21, 26, 27], are required to provide price-hedging mechanism in the spot market, and are several times considered as mechanisms that provide incentives for construction of remote generations projects and merchant transmission investments [22].

The reminder of this paper is organized as follows: section 2 of this paper summarizes the TCUC model and its solution through LR where transmission network security constraints are represented by a DC power flow model; section 3 describes the proposed non-linear pricing alternative for the transmission constrained model in order to cope with nodal prices in disequilibrium resulting from dual variables at the solution of the LR algorithm; In section 4 examples are shown that describe the need for non-liner pricing in transmission constrained unit commitment models, and the solution through the proposed non-linear pricing alternative, along with the application of the redefined FTR’s in the new piece scheme. Finally, section 5 gives conclusions and final remarks.

2. GENERAL TCUC AND LAGRANGIAN RELAXATION

In this section the TCUC problem is presented and its solution through LR is outlined. The TCUC is, in general, a large non-linear mixed-integer, and therefore non-convex, mathematical programming problem who's NP-completeness can be easily proven [13]. The type of transmission model included in the TCUC model considerably affects the complexity of the model, in this paper, as generally accepted for different pricing purposes [1], a linear model is used - DC model as known in the electric power literature [23]. The TCUC model can be written as follows:

$$\text{Min} \quad \sum_{i=1}^{n} C_i(p_i) \tag{1}$$

Subject to:

$$\sum_{i=1}^{n} p_i = p_d \tag{2}$$

$$p_i \in D_i \qquad \forall_i \tag{3}$$

$$p_i = q_i \qquad \forall_i \tag{4}$$

$$\forall q_i \in S \tag{5}$$

where, $C_i(p_i)$ represent generator's power production cost function generally assumed quadratic: $C_i(p_i) = \alpha + \beta p_i + \gamma p_i^2$. Eq. (2) represents the system- wide generation (p_i) - demand (p_d) power balance; (3) represents the operational limits of each generation unit, ramp rates, minimum and maximum output, minimum on and off times; (4) is a constraint whose objective is to duplicated variables p_i in q_i so that transmission constrain in (5) can be decomposed in the LR algorithm. s represents the linear feasible region that represents network constraints. The Lagrangian relaxation solves the dual problem to the primal TCUC problem (1)-(5):

$$\underset{\lambda, s_i}{Max} \quad \psi(\lambda, s_i) \tag{6}$$

where, the dual function is given by: $\psi(\lambda, s_i) = \underset{p_i \in D_i, \forall q_i \in S}{Max} L(\lambda, s_i)$ and the Lagrangian function is given by:

$$L(\lambda, s_i) = \sum_{i=1}^{n} C_i(p_i) + \lambda\left(p_d - \sum_{i=1}^{n} p_i\right) + \sum_{i=1}^{n} s_i(p_i - q_i) \tag{7}$$

where, λ is the dual variable related to the power balance constraint (2) , and s_i are the duals to the variable duplication constraint (3). Using (7), the dual function in (6) is separable in a by-generator profit π_i maximization sub-problems and one transmission sub problem, as follows:

$$\varphi(\lambda, s_i) = \sum_{i=1}^{n} \left[C(p_i) - \lambda p_i + s_i p_i\right] - \underset{\forall q_i \in S}{min} \sum_{i=1}^{n} s_i q_i = -\sum_{i=1}^{n} \pi_i - \underset{\forall q_i \in S}{min} \sum_{i=1}^{n} s_i q_i \tag{8}$$

Using the linear representation of the transmission constraints S [23], the transmission sub problem $\underset{\forall q_i \in S}{Min} \sum_{i=1}^{n} s_i q_i$ in last equation can be written as:

$$Min \sum_{i=1}^{n} s_i q_i \tag{9}$$

Subject to:

$$q = \mathbf{p_d} + \mathbf{B}'\delta \tag{10}$$

$$-\overline{\mathbf{p}}_{ij} \le \mathbf{X}\delta \le \overline{\mathbf{p}}_{ij} \tag{11}$$

where, (10) represents the nodal power balance constraints, $\mathbf{B}'$ is the susceptance matrix and $\mathbf{B}'\delta$ the vector of nodal voltage angles [23]. Eq. (11) represents the transmission lines capacity limits, expressed using linear relation by the node-branch reactance matrix $\mathbf{X}$. Since the objective function in (9) is linear, oscillatory behavior can be expected when solving the dual problem (6) as noted in [9], in order to avoid such situation the nonlinear penalty function (12) is added to (9):

$$\wp = \frac{c}{2}\left(p_i - q_i\right)^2 \tag{12}$$

where, $c/2$ is a positive constant, whose election is made so that it favors the convergence when solving (6) through a non-differential optimization technique such as the sub-gradient method. However, the addition of the term (12) does not allow for separation of the dual function as achieved in (8); the Auxiliary Problem Principle (APP) [24], is used to linearize (12) as follows:

$$\wp = c\left(p_i - q_i\right)\left(p_i^{k-1} - q_i^{k-1}\right) + \frac{b}{2}\left[\left(p_i - p_i^{k-1}\right)^2 + \left(q_i - q_i^{k-1}\right)^2\right] \tag{13}$$

where, p_i^{k-1} and q_i^{k-1} represent the value of variables p_i and q_i in the previous iteration while maximizing the dual function (6) through the sub-gradient method. With this linearization the dual function can still be separated in a per-generator sub-problem and a transmission sub-problem basis:

$$\varphi(\lambda, s_i) = \sum_{i=1}^{n}\left[C(p_i) - \lambda p_i + s_i p_i + \left(cp_i^{k-1} - cq_i^{k-1} - bp_i^{k-1}\right)p_i + \frac{b}{2}p_i^2\right]$$
$$- \min_{q_i \in S_i}\left[\sum_{i=1}^{n} s_i + \left(cp_i^{k-1} - cq_i^{k-1} - bq_i^{k-1}\right)q_i + \frac{b}{2}q_i^2\right] \tag{14}$$

The new transmission sub-problem is as follows:

$$min\left[\sum_{i=1}^{n} s_i + \left(cp_i^{k-1} - cq_i^{k-1} - bq_i^{k-1}\right)q_i + \frac{b}{2}q_i^2\right] \tag{15}$$

Subject to:

$$q = \mathbf{p_d} + \mathbf{B}'\delta$$

$$-\overline{\mathbf{p}}_{ij} \leq \mathbf{X}\delta \leq \overline{\mathbf{p}}_{ij}$$

The gradient vector of the dual function (14), which is used as a maximization direction to solve problem (6) in an iterative fashion is:

$$\partial\varphi(\lambda, s_i) = \begin{bmatrix} \sum_{i=1}^{n} p_{di} - \sum_{i=1}^{n} p_i \\ p_i - q_i \end{bmatrix} \tag{16}$$

Details of the sub-gradient and other several methods to solve the dual problem (6) can be found in [16], [29]. At the solution of the dual problem (6) the dual variable λ represents a market clearing price only if particular conditions are meet [7] (duality gap equal to zero). When an equilibrium price does not exist other pricing alternatives must be studied as pointed in [7] and later in [18]. When transmission constraints are active, the dual variables to the nodal balance power constraints in (15) will represent the locational prices; however, under the same circumstances, these nodal prices as well will not clear the transmission constrained market; therefore, a type of non-linear pricing (second-best option [3]) will be needed to define such locational prices; the proposed coordination methodology to do so is presented in the next section.

3. NON-LINEAR PRICING FOR MARKET CLEARING IN TRANSMISSION CONSTRAINED UNIT COMMITMENT

The pricing alternative developed in this section is based on price coordination which acts as profit redistributors when the prices obtained at the solution of the UC problem do not clear the market. In [7] a simplified price-adjustment produce is proposed to do so, in [18] the first coordination procedure to solve the same problem is proposed for a single-node unit commitment model. In this section we use a simplification of the coordination procedure in [18] to non-linear price the solution to the transmission constrained case. To illustrate the procedure consider a simplified version of a UC model:

$$f^* = \min \sum_{i=1}^{n} c_i(u_i, p_i) \tag{17}$$

Subject to:

$$\sum_{i=1}^{n} p_i = p_d \tag{18}$$

$$u_i \underline{p}_i \leq p_i \leq u_i \overline{p}_i \tag{19}$$

$$p_i \in P_i, \qquad \forall i \tag{20}$$

Let s_i^* be the solution to the dual problem (6), then s_i^* represents a clearing price of the energy market if the individual profit maximization problem (17) leads to the same non-negative profit solution $(\pi(s_i^*) < 0)$ obtained in (6).

$$\pi\left(s_i^*\right)= s_i^* p_i^* - C\left(u_i^*, p_i^*\right) \tag{21}$$

When this condition is not met (the duality gap is also non-zero), the market is not cleared, and a new pricing scheme may be needed; the price coordination mechanism to find such new prices redefines each generation sub-problem as follows:

$$\pi\left(s_i^*\right)= \max_{p_i \in P_i}\left[s_i^* p_i - D\left(u_i, p_i\right) - C_i\left(u_i, p_i\right)\right] \tag{22}$$

where, $D\left(u_i, p_i\right)= \Delta\beta_i p_i$ is the disincentive function associated with the *i-th* producer and can be seen as an additional cost (positive or negative) associated to the generation output [18]. Which is, under disequilibrium, the coordination mechanism is so that the profits of some agents are increased while others are decreased in a zero-sum fashion:

$$\sum_{i=1}^{n} D\left(u_i^*, p_i^*\right)= \sum_{i=1}^{n}\left(\Delta\beta_i p_i^*\right)= 0 \tag{23}$$

The parameters $\Delta\beta_i$ are calculated using the following mathematical program:

$$(AP) \qquad \min \sum_{i=1}^{n}\left[\Delta\beta_i p_i - s_i^* p_i\right] \tag{24}$$

Subject to:

$$\sum_{i=1}^{n}\left(\Delta\beta_i p_i\right)= 0 \tag{25}$$

$$s_i^* p_i^* - C_i\left(u_i^*, p_i^*\right) - D_i\left(u_i^*, p_i^*\right) \geq 0; \qquad i = 1,\ldots,n \tag{26}$$

To sum up, the augmented term in (22) makes the function of a profit-adjusted in the market when the dual variables obtained at the solution of do not represent market clearing prices. The general algorithm to incorporate non-linear pricing in TUCU model is as follows:

Augmenting pricing steps

Step 1: Solve (TCUC) to obtain the schedule $\left(u_i^*, p_i^*\right)$ of each participating producer (generator's), as well as the nodal price, (s_i) obtained as a by-product of solving (TCUC) algorithm.

Step 2: solve the profit maximization problem (21), with the parameters $\left(s_i^*\right)$, to obtain the profits of each producer (generator's).

Step 3: IF $\left(\pi\left(s_i^*\right)\geq 0\right)$ for all i, a competitive market equilibrium has been obtained, which satisfies the profits optimally criterion. Stop.

Step 4: ELSE, form and solve (AP) (24)-(26) to obtain the simplified coordinated price parameter $\Delta\beta_i$, as well as the real price ρ_{arg}, that everything producer and consumer for the electricity pay. Market equilibrium has been found. END

4. NUMERICAL EXAMPLES: NON-LINEAR PRICING FOR TCUC

This section presents a series of examples to illustrate, both, the need for non-linear pricing in transmission constrained unit commitment markets and also the application of the proposed NLCSP methodology developed in this paper, at the same time the need to re-define congestion rents in order continue providing FTR's needed in nodal market designs to provide price hedging is illustrated.

4.1 Numerical example: under equilibrium, nodal linear prices clear the market

The data for an auction with four generators with linear, quadratic but not start-up costs are presented in Table 1. The minimum output power for all the generators is considered zero, the transmission network is presented in Fig. 1, and the transmission capacity is considered 100 MW. The results of the TCUC model including prices and profits are presented in Table 2.

Table 1 Four bidders with quadratic cost functions

i	1	2	3	4
$p_i(MW)$	100	200	50	50
$\beta_i(\$/MW)$	10	30	15	20
$\gamma_i(\$/MW^2)$	0.1	0.8	0.2	0.3

Table 2 Optimal dispatch, unit costs, nodal prices, income and profits

p_d	i	p_i^*	c_i	ρ	$\rho \times p_i^*$	π_i
270	1	81.36	1475.54	26.27	2137.33	661.78
	2	150.00	22500.00	270.00	40500.00	18000.00
	3	28.18	581.52	26.27	740.29	158.77
	4	10.45	241.76	26.27	274.52	32.76

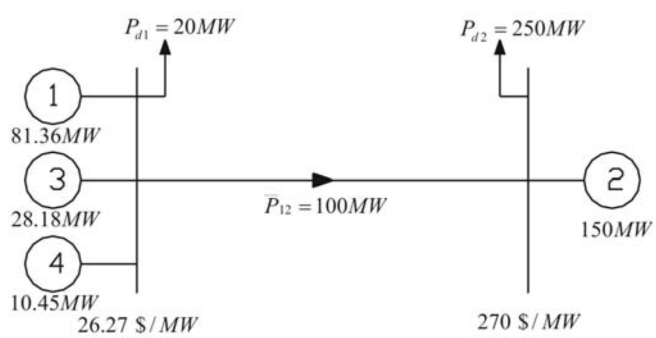

Fig. 1 Optimal dispatch, and nodal prices in equilibrium

As can be seen, due to the congestion in the transmission line, the prices result in 26.27 $/MW at node 1 and 270 $/MW at node 2; the later corresponding to the marginal cost of generator 2. The congestion is preventing the export of cheaper power from node 1 to node 2. These prices are correct pricing signals, or equilibrium prices, since all generators are operating are positive profits and supply and demand are at balance in every location. Under this situation, single nodal prices for each location, the value of the congestion rents (CR_i) can be estimated straightforward by:

$$(CR_i) = (p_j - p_i) \times p_{ij} \tag{27}$$

That is, $100 \times (270\text{-}26.27)$ = $ 24, 733. In general, congestion rents need be estimated as the difference between load payments and generators income at nodal prices.

$$CR = \sum \rho_i p_{di} - \sum \rho_j p_j \tag{28}$$

Which leads to the same valuation of congestion rents as can be obtained from Table 3.

Table 3 Payments: (by) loads and (to) generators

Supplier	Retribution	Demand	Payments
1	$ 2137.33	1	$ 525.40
2	$ 40500.00	2	$ 67500.00
3	$ 740.29		
4	$ 274.52		
Total	*$ 43652.14*		*$ 68025.40*

Assuming an allocation of Z MW in FTR's is awarder to load at node 2, as a mean to mitigate it nodal price, the equivalent, or mitigated, price as seen by this load is:

$$\rho_j^{eq} = \left[\frac{p_d \times \rho_j - Z(\rho_j - \rho_i)}{p_d} \right] \tag{29}$$

If the FTR is by the full transmission capacity, Z=100 MW, the price that load 2 would observe, obtained through the Eq. (29), is only 172.51 $/MW, and if the FTR were by only half of the CR (50 MW), demand 2 would observe it price mitigated only to 221.25 $/MW from its 270 $/MW original price. Nodal prices have the advantage of giving efficient locational price signals, however their variation due to congestion may be perceived by some agents as volatile, FTR's are a mechanism to mitigate such price "volatility" as presented in the latter example when awarding the FTR to load 2.

The way FTR's are awarded considerably varies in different regions, they can be awarded to boat loads, and generators, through an administered procedure, or trough a market procedure (FTR auctions) or combinations of both, we recommend the reader to consult [25-27] in order to board the review issues regarding FTR's design variations (in this examples we uses point-to-point FTR's) and assignment.

4.2 Numerical example: non-linear prices needed to clear the market

In the latter example the nodal prices obtained cleared the market and linear prices where enough, however similar to the unconstrained case (single node UC), these linear prices may not exist for each node; in such circumstances non-linear prices need be defined. Consider the example with four bidders with startup and linear costs shown in Table 4 and connected in the single transmission line system in Fig. 2. The solution to the TCUC leads to the dispatch, costs, prices and profits shown in Table 5.

Table 4 Three bidders with linear (β_i) and startup (α_i) costs

i	1	2	3
$\overline{p}_i(MW)$	150	150	100
$\alpha_i(\$)$	100	150	130
$\beta_i(\$/MW)$	10	20	15

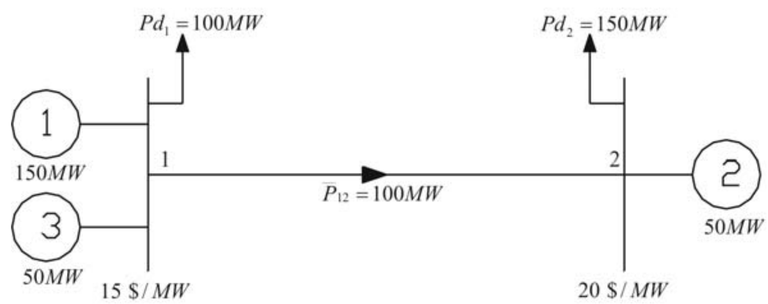

Fig. 2 Optimal dispatch and nodal prices that do not clear the market

Table 5 Optimal solution to the generation dispatch with nodal prices

p_d	i	p_i^*	c_i	ρ	$\rho \times p_i^*$	π_i
250	1	150.00	1600.0	15.00	2250.00	650.00
	2	50.00	1150.0	20.00	1000.00	-150.00
	3	50.00	880.0	15.00	750	-130.00

The transmission line limit of 100 MW leads again to different nodal prices of 15 \$/MW and 20\$/MW at each node, however these prices do not clear the market since generators 2 and 3 are operating at negative profits. This example shows that also in transmission constrained UC models equilibrium prices may not exist and other type of pricing is needed to cope with the situation. After applying the proposed NLCSP algorithm to obtain non-linear prices, the new profits for generation agents are corrected as presented in Table 6.

Table 6 Agents non-linear prices (ρ_i) that reach market equilibrium

Load			Generation				
i	P_d	ρ_i	p_i^*	c_i	ρ	$\rho \times p_i^*$	π_i
1	100	15.65	150.00	1600.00	14.57	2185.50	585.50
2	150	21.00	50.00	1150.00	23.00	1150.00	0.00
3	-	-	50.00	880.00	17.60	880.00	0.00

The new non-linear prices require that each agent receive a different price, for instance not all agents connected to node 1 (load, generators 1 and 3) receive the previous price of 15 \$/MW bur rather they receive a different price of 15.65, 14.57 and 17.60 \$/MW, respectively. Similarly load and generator at node 2 receive a different price each of 21 and 23 \$/MW, respectively. These prices however, coordinate the market, that is, they represent a second-best choice since linear prices that clear the market do not exist. Since there are not single nodal prices for each location, the point-to-point valuation of congestions rents in (27) is no longer valid. Rather, the generalized valuation of congestion rents (28) needs always be used, which in this case leads to a total of $499,40 in congestion rents, as can be obtained from Table 7.

Table 7 Payments: (by) loads and (to) generators

Supplier	Retribution	Demand	*Payments*
1	$ 2185.50	1	$ 1565.00
2	$ 1150.00	2	$ 3150.00
3	$ 880.00		
Total	*$4715.00*		*$4215,50*

If an FTR is awarded to load in node 2 of the system, the point-to-point definition (29) in previous example of the equivalent prices is no longer valid since there are no single prices for each node. Rather the FTR's need be awarded considering each agents particular non-linear price. For instance if an FTR of K MW's is awarded to load in node 2, its equivalent, mitigated, price will be given by the expression:

$$\rho_j^{eq} = \frac{\rho_j \times p_d - K \times CR}{p_d} \tag{30}$$

Under this situation if an FTR from 20 to 100 MW is awarded to load in node 2, the equivalent price this load sees is as shown in Table 8.

Table 8 Different levels of FTR´s and its effect on price mitigation

$K(MW)$	20	40	80	80	100
ρ_j^{eq}	20.33	19.67	19.00	18.34	17.67

4.3 Application of non-linear pricing to the IEEE 14 nodes system

In the single transmission line examples before presented the need for non-linear pricing and the application of the proposed NLCSP can be easily visualized. In this section an application example to the IEEE 14 nodes system is presented, all the data is shown in Appendix. The solution to the TCUC problem leads to the dispatch, cost prices and profits in Table 9.

Table 9 Optimal solution to the generation dispatch with nodal prices

j	p_d	ρ_{nodal}	$p_d \times \rho_{nodal}$	i	p_i^*	c_i	$\rho_{nodal} \times p_i^*$	π_i
1	400.00	8.85	3540.61	1	12.00	334.60	1206.66	872.06
2	250.00	11.67	2916.81	2	20.00	873.58	2011.10	1137.53
3	450.00	8.72	3923.69	3	100.00	2079.90	5248.72	31678.82
4	100.00	11.36	1135.92	4	12.00	336.33	634.37	298.04
5	75.00	23.21	1740.89	5	76.00	1144.83	4017.66	2872.82
6	50.00	26.33	1316.41	6	0.00	0.00	0.00	0.00
7	90.00	14.53	1307.75	7	0.00	0.00	0.00	0.00
8	125.00	14.53	1816.33	8	22.46	628.03	412.77	-215.26
9	250.00	16.74	4185.51	9	6.98	207.13	183.75	-23.38
10	75.00	18.37	1378.06	10	400.00	3610.84	6696.81	3085.97
11	40.00	22.06	882.43	11	66.11	1003.06	960.66	-42.40
12	100.00	52.86	5286.39	12	63.19	963.35	918.23	-45.12
13	75.00	100.56	7541.64	13	0.00	0.00	0.00	0.00
14	150.00	52.49	7873.08	14	337.10	3068.52	2983.89	-84.63
				15	155.00	1910.82	2252.24	341.42
				16	61.32	821.73	696.56	-125.17
				17	2.28	313.18	53.02	-260.16
				18	268.54	3202.01	3133.12	-68.89
				19	322.95	2927.73	2815.89	-111.84
				20	304.05	2778.13	2651.12	-127.02

The transmission lines limits are show in the Table 10. As can be seen from the differences in nodal prices in Fig. 3 there is congestion in the system, however these prices don not clear the market since generators 8-12, 14, 16-20 are operating al negative profits as shown in Fig. 4. After applying NLCSP to obtain non-linear prices the profits for generation agents are correct as present in Table 11.

Table 10 Transmission lines limits $(\bar{p}_{ij})$ from send (N_S) to reception (N_R) node

N_S	N_R	$\bar{p}_i^*$	N_S	N_R	$\bar{p}_i^*$
1	2	80	5	6	*95*
2	3	100	6	13	*35*
3	4	77	7	8	*50*
4	5	120	9	10	*90*

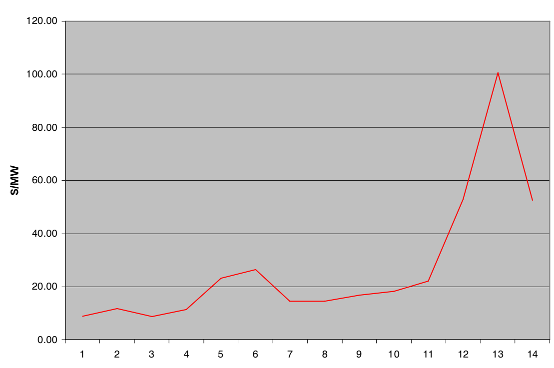

Fig. 3 Nodal prices of energy for the 14 nodes system

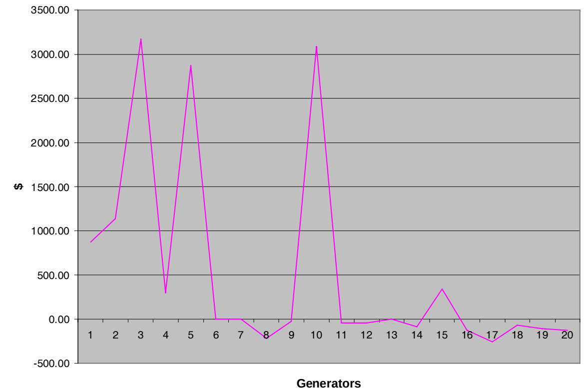

Fig. 4 Generators profits with nodal prices (negatives profits exist)

Table 11 Agents non-linear prices (ρ_{arg}) that reach market equilibrium

Load				Generation					
j	p_d	ρ_{arg}	$p_d \times \rho_{arg}$	i	p_i^*	c_i	ρ_{arg}	$\rho_{arg} \times p_i^*$	π_i
1	400.00	9.10	3639.61	1	12.00	334.60	97.15	1168.79	831.19
2	250.00	11.91	2978.68	2	20.00	873.58	97.89	1957.79	1084.22
3	450.00	8.97	4035.06	3	100.00	2079.90	51.00	5100.21	3020.31
4	100.00	11.61	1160.67	4	12.00	336.33	51.70	620.40	284.07
5	75.00	23.46	1759.45	5	76.00	1144.83	51.09	3883.02	2738.19
6	50.00	26.58	1328.79	8	22.46	628.03	27.96	628.03	0.00
7	90.00	14.78	1330.03	9	6.98	207.13	29.68	207.13	0.00
8	125.00	14.78	1847.26	10	400.00	3610.84	16.38	6552.18	2941.34
9	250.00	16.99	4247.38	11	66.11	1003.06	15.17	1003.06	0.00
10	75.00	18.62	1396.62	12	63.19	963.35	15.24	963.35	0.00
11	40.00	22.31	892.33	14	337.10	3068.52	9.10	3068.52	0.00
12	100.00	53.11	5311.14	15	155.00	1910.82	14.43	2236.24	325.42
13	75.00	100.80	7560.20	16	61.32	821.73	13.40	821.73	0.00
14	150.00	52.73	7910.20	17	2.28	313.18	137.11	313.18	0.00
				18	268.54	3202.01	11.92	3202.01	0.00
				19	*322.95*	*2927.73*	*9.07*	*2927.73*	0.00
				20	304.05	2778.13	9.14	2778.13	0.00

The new set of non-linear prices require that each agent receive a different price for the energy, for instance no all agents connected to node 13 (load, generator 1 and 2) receive the previous price nodal of 100.56 $/MW bur rather they receive a different price of 100.8, 97.15 and 97.89 $/MW, respectively. For the rest of the system applies similar, there are as many prices as suppliers and consumers connected to a node. The new set of prices, however, coordinates the market, that is, they represent a second-best choice since equilibrium linear prices do not exist. In Fig. 5 a comparative chart between linear nodal and augmented prices is shown. In Fig. 6 a comparative of generators profits with linear and non-linear pricing, as can be seen non-linear pricing acts as a profit redistributors to avoid loses of some of the generators.

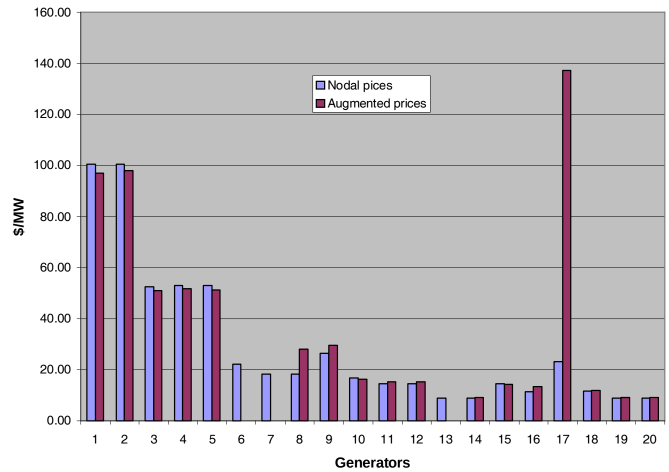

Fig. 5 Nodal prices & augmented prices

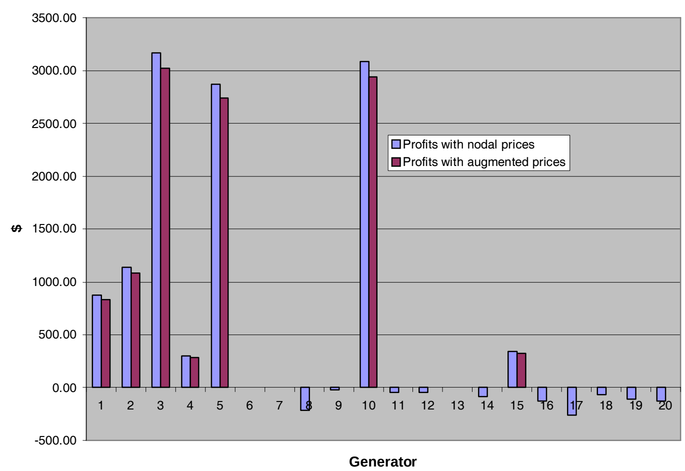

Fig. 6 Profits with nodal and augmented prices

Congestion rents computed from (28) add up to $ 7968.93. Assuming that demands at nodes 12-14 are being affected by their prices, the provision of FTR's to these load can be a mechanism to mitigate such prices. Since non-linear prices where required in the market FTR´s need be awarded considering each agent particular non-linear price. For instance if an FTR of K MW´s is Fig. 7 for different levels 0, to 100% of FTRs, distributed equally to each load.

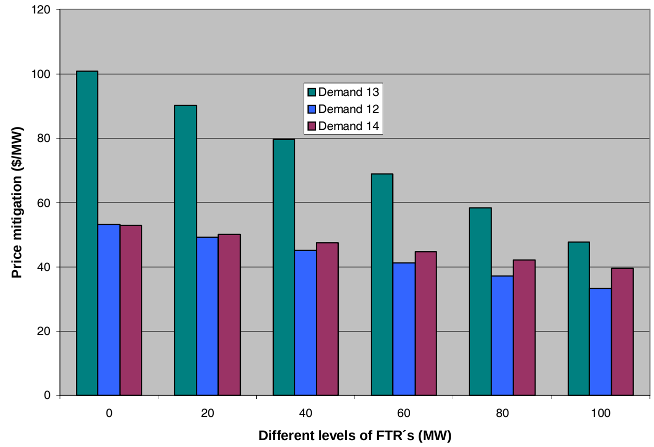

Fig. 7 Different levels of FTR´s (0-100%), and the effects on price mitigation at nodes 12-14

5. CONCLUSIONS

Several electricity markets use transmission constrained unit commitment models as the clearing mechanisms for the spot market for energy. The existence of equilibrium prices in non constrained unit commitment models has been recently investigated, however such situations can also appear in transmission constrained market as shown in this paper, which means, that under particular situations, nodal (or locational) prices may also not clear the market. Based on recent research on non-linear pricing for single-node unit commitment models, this paper proposes a nodal non-linear pricing alternative based on coordination functions inside a Lagrangian relaxation algorithm to solve the transmission-constrained unit commitment models. The new set of non-linear prices coordinates and clears the market but leads to as many prices as agents connected to each node in the system. Since unique nodal prices do not longer exist, the traditional point-to-point definition of Financial Transmission Rights is revised and redefined for its application with the new non-lineal pricing methodology.

6. ACKNOWLEDGEMENTS

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8 APPENDIX

Fig. 6 shows the 14 node system used in the example, generators data Table 12, and nodal load data in Table 13.

Table 12 Twenty bidders with linear (β_i), quadratic (γ_i^2) and startup (α_i) costs

i	*Node*	$\alpha_i(\$)$	$(\beta_i \$/MW)$	$\gamma_i(\$/MW^2)$	$\bar{p}_i(MW)$
1	13	24.3891	25.5472	0.0253	12.0
2	13	117.7551	37.5510	0.0120	20.0
3	14	217.8952	18.0000	0.0062	100.0
4	12	24.4110	25.6753	0.0265	12.0
5	12	81.1364	13.3272	0.0088	76.0
6	11	24.6382	25.8027	0.0280	12.0
7	10	24.8882	26.0611	0.0286	12.0
8	10	218.3350	18.1000	0.0061	100.0
9	6	24.7605	25.9318	0.0284	12.0
10	9	310.0021	7.4921	0.0019	400.0
11	8	81.2980	13.3538	0.0089	76.0
12	8	81.4641	13.3805	0.0091	76.0
13	1	177.0575	10.8616	0.0015	350.0
14	1	311.9102	7.5031	0.0020	400.0
15	7	142.7348	10.6940	0.0046	155.0
16	4	143.5972	10.7583	0.0049	155.0
17	5	260.1760	23.2000	0.0026	197.0
18	2	177.0575	10.8616	0.0015	350.0
19	3	310.0021	7.4921	0.0019	400.0
20	3	311.9102	7.5031	0.0020	400.0

Table 13 Nodal Demands (MW)

i	*Node*	$\bar{p}_i(MW)$	i	*Node*	$\bar{p}_i(MW)$
1	1	400	8	8	125
2	2	250	9	9	250
3	3	450	10	10	75
4	4	100	11	11	40
5	5	75	12	12	100
6	6	50	13	13	75
7	7	90	14	14	150

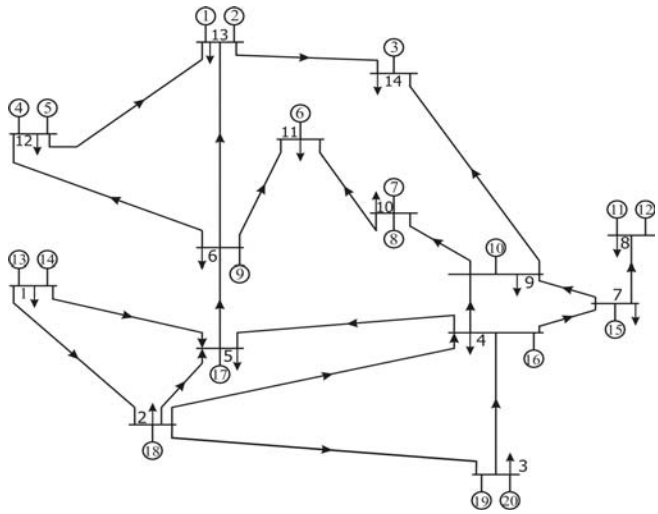

Fig. 6 14 nodes system for the TCUC problem