Optimal Placement of Multi-Type FACTS Devices by Simulated Annealing Approach

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P. Bhasaputra and W. Ongsakul
Energy Field of Study
School of Environment, Resources and Development,
Asian Institute of Technology, Pathumthani 12120
THAILAND

ABSTRACT

In this paper, a simulated annealing (SA) approach is proposed to find the optimal placement of multi-type FACTS devices to minimize the total cost including the total generator fuel cost of all loading levels and the cost of FACTS devices. The problem is decomposed into the optimal placement of multi-type FACTS devices subproblem that is searched by the SA approach and the OPF with multi-type FACTS devices subproblem that is also solved by the hybrid tabu search and simulated annealing (TS/SA) approach and quadratic programming (QP). Four types of FACTS devices are used: thyristor-controlled series capacitor (TCSC), thyristor-controlled phase shifter (TCPS), unified power flow controller (UPFC), and static var compensator (SVC). The solution includes multi-location, multi-type, and multi-size of FACTS devices. Test results on the 6 bus system with multi-type FACTS devices indicate that the proposed SA approach can obtain better solutions than the sensitivity index approach. Moreover, SA approach converges to the optimal solution at a faster rate than genetic algorithm (GA) and tabu search (TS).

1. INTRODUCTION

Flexible AC transmission system (FACTS) devices are integrated in power systems to control power flow, increase transmission line stability limit, and improve the security of transmission systems [1]. FACTS controllers are used to enhance system flexibility and increase system loadability.

Sensitivity index approach was commonly used to determine the optimal placement of FACTS devices. For example, the new performance index was used to determine the optimal location of static var compensator (SVC) based on system loadability and contingency analysis [2]. The single contingency sensitivity (SCS) criterion was used to develop a branch's prioritizing index in order to rank branches for possible placement of thyristor-controlled series capacitor (TCSC) [3]. The loss sensitivity with respect to TCSC and thyristor-controlled phase shifter (TCPS) placed in line *k* has been used to determine the optimal placement of FACTS devices in an electricity market including pool and contractual dispatches [4].

In the advent of heuristic methods, genetic algorithm (GA) was used to search for the optimal placement of unified power flow controller (UPFC) to minimize the total generation cost [5]. All possible locations of UPFC were encoded to the population of GA. The total generation fuel cost was used to evaluate the quality of the solution. Meanwhile, GA was used to find the optimal placement of phase shifters in the French network [6]. The overload transmission lines were selected as the candidates of optimal placement and were encoded to the binary string in the population of GA. The OPF was run for each solution to evaluate the quality by using the METRIS software. GA was also used to determine the location and compensation level of TCSC with the aim of maximizing total transfer capacity (TTC) [7]. TTC was used to evaluate the impacts of TCSC by the continuation power flow subject to line thermal limits and bus voltage magnitude limits. The solution vector included location and compensation level of TCSC.

The heuristic based optimization with FACTS devices problem were considered only one type of FACTS device, but FACTS devices may be installed more than one type in a power system. Thus, GA was applied to search for the optimal location of multi-type FACTS devices including TCSC, TCPST, thyristor-controlled voltage regulator (TCVR) and SVC [8]. The optimizations were performed on three parameters: the locations, types and parameter settings. They were encoded to the solution matrix containing floating point elements. The GA functions were applied to find the optimal solution to maximize the system loadability by treating branch loading and voltage level limits in a soft manner.

This paper proposes FACTS devices to reduce the total cost included cost of FACTS devices. The optimal placement of multi-type FACTS devices problem is decomposed into the optimal placement of multi-type FACTS devices subproblem and the OPF with fixed location of multi-type FACTS devices subproblem. The solutions of first subproblem are the placements of multi-type FACTS devices that are searched by the proposed simulated annealing (SA) approach. The second subproblem is OPF with the fixed location of multi-type FACTS devices from the first subproblem. The hybrid TS/SA approach proposed in [9] is used to search for FACTS parameters and quadratic programming (QP) is used to evaluate the solution quality of OPF with multi-type FACTS devices. The placement solution includes multi-location, multi-type and multi-size of FACTS devices. The optimally placed multi-types of FACTS devices OPF is used to minimize the total cost including the total generator fuel cost and the cost of FACTS devices subject to power balance constraint, real and reactive power generation limits, voltage limits, transmission line limits, and FACTS parameters limits. The proposed SA for optimal placement is tested and compared to the sensitivity index approach and GA on the 6 bus system with multi-type FACTS devices for all loading levels.

2. OPTIMAL PLACEMENT OF MULTI-TYPE FACTS DEVICES

The problem is decomposed into the optimal placement of multi-type FACTS devices subproblem and the OPF with fixed location of multi-type FACTS devices subproblem. The solutions of first subproblem are the placements of multi-type FACTS devices that are searched by the proposed SA approach. The second subproblem is OPF with the fixed location of multi-type FACTS devices from the first subproblem. The hybrid tabu search and simulated annealing (TS/SA) approach proposed in [9] is used to search for FACTS parameters and quadratic programming (QP) is used to evaluate the solution quality of OPF with multi-type FACTS devices.

2.1 Optimal Placement of Multi-type FACTS Devices Problem Formulation

The optimal placement of FACTS devices is formulated as:

$$\text{Min} \quad \mathrm{F}(\mathbf{S}) = \sum_{ll=1}^{NLL} \mathrm{F}(\mathbf{S})_{ll} + \mathrm{F}(\mathbf{F}) \tag{1}$$

where, $$\mathrm{F}(\mathbf{S})_{ll} = \sum_{i \in NG} (a_i + b_i \times P_{Gi,ll} + c_i \cdot P_{Gi,ll}^2) \tag{2}$$

$\mathrm{F}(\mathbf{S})_{ll}$ = The objective function of solution matrix **S** obtained by solving the OPF with multi-type FACTS devices of loading level *ll*,

$\mathrm{F}(\mathbf{F})$ = The total cost of multi-type FACTS devices,

S = The solution matrix of FACTS devices placement,

$P_{Gi,ll}$ = The real power generation at bus *i* of loading level *ll* (MW),
NG = the set of generation bus indices, and
NLL = the number of loading levels.

The solutions of multi-type FACTS devices placement are assigned to OPF with multi-type FACTS devices subproblem, minimizing the total generator fuel cost of each loading level. It is used to evaluate the quality of multi-type FACTS devices placement.

2.2 OPF with Multi-type FACTS Devices Problem Formulation

The FACTS devices parameters are additional control variables that cannot be solved by the conventional OPF because these parameters will change the admittance matrix. Therefore, the OPF with FACTS devices problem is decomposed into two subproblems.

2.2.1 Optimal Setting of FACTS Parameters Subproblem

The hybrid TS/SA approach proposed in [9] is used to determine each optimal setting of FACTS parameters for each loading level within their limits and power flow security limits. FACTS devices control variables will be fixed in the conventional OPF subproblem, which is solved by the quadratic programming (QP). The results from the QP OPF are used to evaluate the quality of FACTS parameters.

2.2.2 OPF with Fixed FACTS Parameters Subproblem

The OPF with fixed FACTS parameters subproblem is formulated as:

$$\text{Min} \qquad \sum_{i \in NG} (a_i + b_i \times P_{Gi} + c_i \cdot P_{Gi}^2) \tag{3}$$

Subject to:

$$(P_{Gi} - P_{Di}) + \sum_{k=1}^{m(i)} P_{Pi}(\alpha'_{Pk}) + \sum_{k=1}^{n(i)} P_{Ui}(|V'_{Uk}|, \alpha'_{Uk}) - \sum_{j=1}^{N} |V_i||V_j||Y_{ij}(\mathbf{X}'_S)| \cos(\theta_{ij}(\mathbf{X}'_S) - \delta_{ij}) = 0 , \quad \forall i \in N \tag{4}$$

$$(Q_{Gi} - Q_{Di}) + \sum_{k=1}^{m(i)} Q_{Pi}(\alpha'_{Pk}) + \sum_{k=1}^{n(i)} Q_{Ui}(|V'_{Uk}|, \alpha'_{Uk}) + Q'_{Vi} + \sum_{j=1}^{N} |V_i||V_j||Y_{ij}(\mathbf{X}'_S)| \sin(\theta_{ij}(\mathbf{X}'_S) - \delta_{ij}) = 0 , \quad \forall i \in N \tag{5}$$

$$P_{Gi,min} \le P_{Gi} \le P_{Gi,max} \qquad \forall i \in NG \tag{6}$$

$$Q_{Gi,min} \le Q_{Gi} \le Q_{Gi,max} , \qquad \forall i \in NG \tag{7}$$

$$|V_i|_{min} \le |V_i| \le |V_i|_{max} , \qquad \forall i \in N \tag{8}$$

$$|Sl_i| \le Sl_{i,\max} , \qquad \forall i \in NL \tag{9}$$

where, P_G = the real power generation at bus *i* (MW),
P_D = the real power demand at bus *i* (MW),
$P_{Pi}(a¢_{Pk})$ = the injected real power of TCPS *k* connected at bus *i* (MW),
$P_{Ui}(|V2_{Uk}|, a2_{Uk})$ = the injected real power of UPFC *k* connected at bus *i* (MW),
Q_{Gi} = the reactive power generation at bus *i* (MVAR),
Q_{Di} = the reactive power demand at bus *i* (MVAR),

$Q_{Pi}(\alpha'_{Pk})$ = the injected reactive power of TCPS k connected at bus i (MVAR),
$Q_{Ui}(|V'_{Uk}|, \alpha'_{Uk})$ = the injected reactive power of UPFC k connected at bus i (MVAR),
Q'_{Vi} = the fixed injected reactive power of SVC at bus i from the first subproblem (MVAR),
$|V_i|$ = the voltage magnitude at bus i,
$|V_j|$ = the voltage magnitude at bus j,
δ_{ij} = the voltage angle difference between bus i and bus j,
Sl_i = the apparent power flow in transmission line i (MVA),
$Y_{ij}(\mathbf{X}'_S)|$ = the magnitude of the ij^{th} element in Y_{bus} with TCSC included,
$\theta_{ij}(\mathbf{X}'_S)$ = the angle of the ij^{th} element in Y_{bus} with TCSC included,
$\mathbf{X}'_S$ = the set of fixed reactance of TCSC from the first subproblem,
α'_{Pk} = the fixed phase shift angle of TCPS k connected at bus i from the first subproblem,
$|V'_{Uk}|$ = the fixed voltage magnitude of UPFC k connected at bus i from the first subproblem,
α'_{Uk} = the fixed voltage angle of UPFC k connected at bus i from the first subproblem,
N = the set of bus indices,
NL = the set of transmission line indices,
$m(i)$ = the number of TCPS connected at bus i, and
$n(i)$ = the number of UPFC connected at bus i,

3. SIMULATED ANNEALING APPROACH FOR OPTIMAL PLACEMENT OF FACTS DEVICES

3.1 Solution Coding

Table 1 gives the FACTS types and their parameters. A solution matrix of FACTS devices placement is shown in Eq. 10. The integer coding is used to represent type, location, and size indices of FACTS devices. Details of solutions are shown in Appendix.

Table 1 FACTS types and their parameters

Type	Type index	Type of connection and location	Parameter	Size	Cost
TCSC	1	Series, location index is line number	X_s	Maximum setting of X_s	Cost of TCSC
TCPS	2	Series, location index is line number	α_p	Maximum setting of α_p	Cost of TCPS
UPFC	3	Series, location index is line number	V_u, α_u	Maximum setting of V_u and α_u	Cost of UPFC
SVC	4	Parallel, location index is bus number	Q_V	Maximum setting of Q_V	Cost of SVC

$$\text{Solution }(\mathbf{S}) = \left[\text{Type index vector}, \text{Location index vector}, \text{Size index vector}\right]_{4\text{x}3} \tag{10}$$

3.2 Initialization

One small size FACTS device of each type is used as an initial solution. The initial location of TCSC and TCPS are obtained by using the loss sensitivity index with respect to FACTS devices placed according to the off-peak loading level. Initial location of UPFC is randomly generated whereas SVC is initially placed at the maximum reactive load bus of the off-peak loading level. Details of them are as follows:

3.2.1 Thyristor-Controlled Series Capacitor

Real power loss of line k (P_{LK}) can be expressed as:

$$P_{LK} = |V_i|^2 G_{ij} + |V_j|^2 G_{ij} - 2|V_i||V_j| G_{ij} \cos(\delta_{ij}) \tag{11}$$

The total real power loss (P_L) in the system is

$$P_L = \sum_{K=1}^{NL} P_{LK} \tag{12}$$

The parameter of TCSC (X_s) can be seen as X_{ij} in the total real power loss equation. The loss sensitivity index with respect to TCSC placed in line k ($\acute{a}_K$) can be given as:

$$\alpha_K = \frac{\partial P_L}{\partial X_{ij}} = \left[|V_i|^2 + |V_j|^2 - 2|V_i||V_j| \cos(\delta_{ij}) \right] * \left[\frac{-2R_{ij} X_{ij}}{\left(R_{ij}^2 + X_{ij}^2\right)^2} \right] \tag{13}$$

The TCSC should be placed in a line that has maximum positive of $\acute{a}_K$.

3.2.2 Thyristor-Controlled Phase Shifter

The parameter of TCPS ($\acute{a}_p$) can be seen as $\acute{a}_{ij}$ in the total real power loss equation. The loss sensitivity index with respect to TCPS placed in line k (b_K) can be given as:

$$b_K = \frac{\partial P_L}{\partial \delta_{ij}} = 2|V_i||V_j| G_{ij} \sin(\delta_{ij}) \tag{14}$$

The TCPS should be placed in a line that has maximum absolute value of b_K.

3.2.3 Unified Power Flow Controller

The initial placement of UPFC is determined by:

$$Location_{UPFC} = u * NL \tag{15}$$

where, u = a uniform randomly generated number between 0 and 1.

3.2.4 Static Var Compensator

SVC is placed at the bus that has maximum reactive load for the off-peak loading level.

3.3 Perturbation

The trial solution matrices are generated by perturbing the initial solution matrix as shown in Fig. 1. The trial feasible solutions do not allow the same type of FACTS devices placed at the same location. The temperature at iteration k is normalized to compare with the uniform randomly generated number between 0 and 1 (u) for perturbation of the solution. Perturbations are more frequent at the beginning of iterations and less frequent due to the smaller temperature at iteration k (T_k) as the iteration grows.

$$T_k = r^{(k-1)} * T_1 \tag{16}$$

where, T_1 = the initial temperature,
T_k = the temperature at iteration k,
k = the iteration counter, and
r = the reduction rate.

3.4 Acceptance Criterion

The acceptance criterion is designed for decision movement of the trial solution. When the probabilistic acceptance criterion is higher than a uniform randomly generated variable in the interval [0,1), the trial solution matrix is set to the initial solution matrix of the next iteration. The probabilistic acceptance criterion is given as follows:

$$p^k = \frac{1}{1+\exp(\Delta/T_k)} \tag{17}$$

where, p^k = the probabilistic acceptance criterion of trial solution matrix at iteration k,
Δ = the difference between the objective function of the trial solution matrix and the best solution matrix reached.

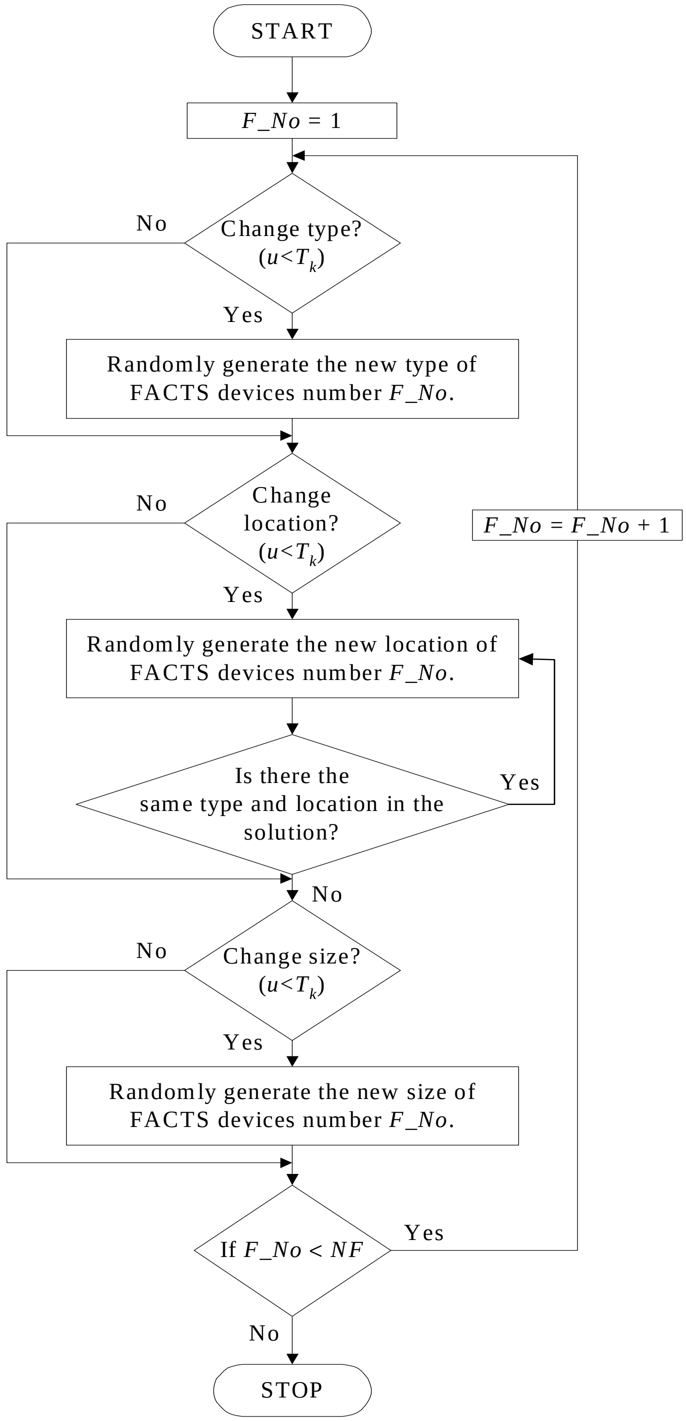

Fig. 1 Perturbation process of SA without fixed number for each type of FACTS devices

where, F_No = the index number of FACTS devices, and
NF = the total number of FACTS devices.

3.5 Simulated Annealing Procedure

The following notations are additionally used for the SA approach.

k_u = the iteration counter without improving the best objective function reached (F_B),
k_T = the trial solution matrix counter,
$\mathbf{S_B}$ = the best solution matrix reached,
$\mathbf{S_T^{(k,0)}}$ = the initial solution matrix at iteration k,
$\mathbf{S_T^{(k+1,0)}}$ = the initial solution matrix at iteration k+1,
$\mathbf{S_T^{(k,m)}}$ = the M trial solution matrix at iteration k, m=1,…,M,
F_B = the best objective function reached,
$k_{u,max}$ = the specified maximum allowable number of iterations without improving F_B,
k_{max} = the maximum allowable number of iterations, and
M = the specified number of trial solution matrices.

The SA procedure for optimal placement of FACTS devices is as follows:

Step 1: Read the system and unit data, FACTS data, and load demand of each loading level.
Step 2: Specify $k_{max} = 50$, $k_{u,max} = 5$, and M = 20
Step 3: Specify $T_1 = 30$ and $r = 0.97$.
Step 4: Set k=1 and k_u=1.
Step 5: Randomly generate $\mathbf{S_T^{(k,0)}}$. Set $\mathbf{S_B} = \mathbf{S_T^{(k,0)}}$ and $F_B = F(\mathbf{S_T^{(k,0)}})$.
Step 6: Determine T_k by Eq. (16).
Step 7: Set k_T=1.
Step 8: Randomly generate trial solution matrix by perturbation process in Section 3.3 to obtain $\mathbf{S_T^{(k,k_T)}}$ and Evaluate $F(\mathbf{S_T^{(k,k_T)}})$.
Step 9: If $F(\mathbf{S_T^{(k,k_T)}}) < F_B$, set $F_B = F(\mathbf{S_T^{(k,k_T)}})$, $\mathbf{S_B} = \mathbf{S_T^{(k,k_T)}}$, $\mathbf{S_T^{(k,0)}} = \mathbf{S_T^{(k,k_T)}}$. Then set k_u=1 and goto Step 11 Otherwise, set $k_u = k_u + 1$.
Step 10: Check the acceptance criterion. If $p^k > u$, set $\mathbf{S_T^{(k,0)}} = \mathbf{S_T^{(k,k_T)}}$.
Step 11: If $k_T < M$, set $k_T = k_T + 1$ and goto Step 8. Otherwise $\mathbf{S_T^{(k+1,0)}} = \mathbf{S_T^{(k,0)}}$.
Step 12: If $k < k_{max}$ and $k_u < k_{u,max}$, set $k = k+1$ and go to Step 6. Otherwise, terminate the process and $\mathbf{S_B}$ is the optimal placement of multi-type FACTS devices.

4. SIMULATION RESULTS

The generation cost functions and topology of the 6 bus system are given in [10]. There are nine case studies. Cases 1-2 are the OPF without FACTS devices without and with line constraints for two loading levels that are used as the base cases. Details of Cases 1-7 are as follows:

Case 1: The OPF without FACTS devices neglecting line flow limit constraints for off-peak and peak loading levels.
Case 2: The OPF without FACTS devices is used as a base case for off-peak and peak loading levels.

Case 3: The optimal placement of multi-type FACTS devices with one small FACTS device of each type for off-peak loading level by using the sensitivity index approach.
Case 4: The optimal placement of multi-type FACTS devices with one small FACTS device of each type for off-peak loading level by the SA approach.
Case 5: The optimal placement of multi-type FACTS devices with one small FACTS device of each type for off-peak and peak loading levels by using the sensitivity index approach.
Case 6: The optimal placement of multi-type FACTS devices with one small FACTS device of each type for off-peak and peak loading levels by the SA approach.
Case 7: The optimal placement of multi-type FACTS devices for off-peak and peak loading levels by the SA approach with any type and size of four FACTS devices.

Table 2 shows the total generator fuel cost and total cost of Cases 1-7. For off-peak loading level, the results show that line flow limit constraint increases the total generator fuel cost from \$3,126.4 to \$3,143.9. When multi-type FACTS devices are installed, the total generator fuel cost is reduced from \$3,143.9 (Case 2) to \$3,118.7 (Case 4). The proposed SA approach can find the better solution than the sensitivity index approach. In single loading level, the total generator fuel cost is reduced from \$3,125.1 (Case 3) to \$3,118.7 (Case 4) or 0.205% and from \$7,718.5 (Case 5) to \$7,695.1 (Case 6) or 0.303% for two loading levels. For Case 7, the total cost is reduced from \$7,735.1 (Case 6) to \$7,718.8. When each type of FACTS devices is not fixed to one, the total cost is lower than one FACTS device of each type. Table 3 shows the solution of Case 7.

Table 2 Simulation results of Cases 1-7

Case	Total cost of load level		Total generator fuel cost (\$)	FACTS cost		*Total cost (\$)*
	Off Peak (\$/hr)	Peak (\$/hr)		Off Peak (\$/hr)	Peak (\$/hr)	
1	3126.4	4541.2	7667.6	0	0	*7667.6*
2	3143.9	4666.5	7810.4	0	0	*7810.4*
3	3125.1	-	3125.1	20	-	*3145.1*
4	3118.7	-	3118.7	20	-	*3138.7*
5	3125.1	4593.4	7718.5	20	20	*7758.5*
6	3125.7	4569.4	7695.1	20	20	*7735.1*
7	*3125.1*	*4565.7*	*7690.8*	*14*	*14*	7718.8

Table 3 The solution of Case 7

Type index	Location index	Size index
1	9	1
1	10	1
2	2	1
2	3	1

There are two more cases to compare the results of the different heuristic method that are GA and TS. The details are as follows:

Case 8: The optimal placement of multi-type FACTS devices for off-peak and peak loading levels by the GA with any type and size of four FACTS devices. The integer coding is used to encode the solution in matrix form as shown in Eq. 10. The Roulette wheel selection and one point crossover are used to generate the offspring by swapping the row of parent matrices as shown in Fig. 2. The mutation process randomly selects the element of offspring matrix and also randomly generates the new value of selected element of solution index by:

$$\text{Value}_{\text{new}} = [\, u * \text{Upper limit of selected element of solution index} \,] \qquad (18)$$

$$\text{Parent 1} \quad \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{bmatrix} \quad \text{Offspring 1} \quad \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 3 & 6 & 1 \\ 4 & 3 & 1 \end{bmatrix}$$

$$\text{Parent 2} \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 3 & 6 & 1 \\ 4 & 3 & 1 \end{bmatrix} \quad \text{Offspring 2} \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{bmatrix}$$

Fig. 2 The one point crossover at 3rd row

After crossover or mutation processes, if the offspring solutions include the same type of FACTS devices placed at the same location, the crossover or mutation process will be repeated to avoid placing the same FACTS type at the same location. The parameters of GA are set as: maximum number of iterations = 50, crossover probability = 0.7 and mutation probability = 0.01.

Case 9: The optimal placement of multi-type FACTS devices for off-peak and peak loading levels by TS with any type and size of four FACTS devices. The trial neighborhood solution matrices are generated by using perturbation process in Fig. 1 with fixed T_k at 0.5 for all iteration. Aspiration level (AL) is set to the total cost of the current neighborhood solution vector at the previous iteration ($F(\mathbf{S}_{\mathbf{T}}^{(\mathbf{k-1,0})})$). If $\mathbf{S}_{\mathbf{T}}^{(\mathbf{k,m})}$ is in tabu list and $F(\mathbf{S}_{\mathbf{T}}^{(\mathbf{k,m})}) < AL$, $\mathbf{S}_{\mathbf{T}}^{(\mathbf{k+1,0})} = \mathbf{S}_{\mathbf{T}}^{(\mathbf{k,m})}$ and AL is updated by $F(\mathbf{S}_{\mathbf{T}}^{(\mathbf{k,m})})$.

Table 4 shows the best solutions and their CPU times of Cases 7-9. Table 5 gives the comparison of TS and GA from 10 runs. For the TS can search the same solution for all runs but GA cannot found the same solution for all runs Furthermore, the average CPU time of the SA is 29.51% and 9.71% less than GA and TS, respectively. More specifically, the SA converges at a faster computing time than GA and TS as shown in Fig. 3.

Table 4 Simulation results of Cases 7-9 best solutions

Case	Total generator fuel cost ($)	Total cost of FACTS devices ($)	Total cost ($)	CPU time (hours)
7	7690.8	28	7718.8	8:50
8	7690.8	28	7718.8	10:30
9	7690.8	28	7718.8	9:25

Table 5 Comparison of TS and GA from 10 runs (Total cost ($))

	Worst	Average	Best	Average CPU time (hours)
Case 7	7718.8	7718.8	7718.8	9:26
Case 8	7733.1	7725.4	7718.8	12:13
Case 9	7718.8	7718.8	7718.8	10:21

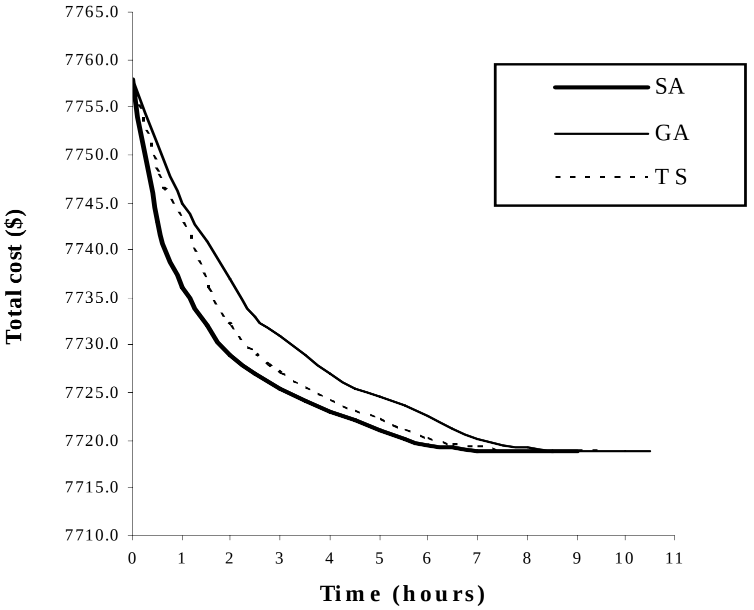

Fig. 3 The convergence comparison of best solutions

5. CONCLUSIONS

In this paper, the SA approach is efficiently used to find the optimal placement of multi-type FACTS devices to minimize the total cost of off-peak and peak loading levels in the 6 bus power system. The SA approach achieves better solutions and requires less CPU times than GA and TS. Multi objectives including stability improvement will be investigated in our future research work.

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7. APPENDIX

Table 6 Details of solutions

Type	Type index	Set of location	Size of FACTS by parameters (size index)	Cost of FACTS ($/hr)
TCSC	1	{1, 2, ..., 11}	0.15 pu. (1)	3
			0.30 pu. (2)	6
TCPS	2	{1, 2, ..., 11}	±1.0 radian (1)	4
			±2.0 radian (2)	8
UPFC	3	{1, 2, ..., 11}	V_u 0.1 pu. (1)	10
			V_u 0.2 pu. (2)	15
SVC	4	{1, 2, .., 6}	2.5 MVar (1)	3
			5.0 MVar (2)	6

Note: Low limit of X_s, V_u and Q_V are set at 0.
Limit of α_u is set at $\pm\pi$.

The investment cost per hour of FACTS devices can be expressed as follow:

$$\text{Total investment cost per hour of small FACTS devices} = 0.005 * \text{average total generator fuel cost of off-peak and peak loading level of Case 2} \quad (19)$$

The total investment cost per hour of small FACTS devices is $40. The investment cost per hour of each small FACTS device is 15%, 20%, 50% and 15% for TCSC, TCPS, UPFC and SVC, respectively. The investment cost per hour of each large FACTS type is twice of the investment cost per hour of each small FACTS type, except the investment cost per hour of large UPFC is 150% of investment cost per hour of small UPFC. If the life time of FACTS devices is assumed to be 15 years, the initial investment cost of FACTS devices are shown Table 7.

Table 7 Details of investment cost of FACTS devices

Type	Life time (years)	Size	*Total cost ($)*
TCSC	15	small	*394,200*
		large	*788,400*
TCPS	15	small	*525,600*
		large	*1,051,200*
UPFC	15	small	*1,314,000*
		large	*1,971,000*
SVC	15	small	*394,200*
		large	*788,400*