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Tracing Reactive Power Flow and Loss in Electric Power Systems

Mohd Herwan Sulaiman*[1], Mohd Wazir Mustafa[+], Omar Aliman[#], Ismail Daut*, Surya Hardi Amrin* and Mohamad Suhaizal Abu Hassan^

Abstract – *This paper suggests a systematic method to trace the reactive power flow and loss named as Proportional Tree Method (PTM). From the power flow solution, the test system is modeled conceptually like a tree, where the reactive power flow tracing is started from a particular generator to a particular line or load through the routes that connect between them. It is also possible to pinpoint the loss at each transmission line to which generator. The veracity and simplicity of the method is demonstrated by numerical examples.*

Keywords – Deregulation, power flow tracing, proportional sharing principle, proportional tree method, transmission open access.

1. INTRODUCTION

The competitive environment of electricity markets necessitates wide access to transmission and distribution networks that connect dispersed customers and suppliers. Moreover, as power flow influence transmission charges, transmission pricing may not only determine the right of entry but also encourage efficiencies in power markets. A proper transmission pricing scheme that considers transmission constraints or congestion could motivate investors to build new transmission and/or generating capacity for improving the efficiency [1].

Regardless of the market structure, it is important to accurately determine transmission usage in order to implement usage-based cost allocation methods. However, determining an accurate transmission usage could be difficult due to nonlinear nature of power flow. To overcome this matter, approximate models and tracing algorithms are proposed to allocate the contributions of individual generators to transmission lines, loads and losses. This paper emphasizes on the reactive power-flow tracing algorithm due to the fact that the transmission open access may also require pricing of reactive power transmission.

Several methods of power-flow tracing are already proposed in the literatures [1]-[10]. The method proposed in [4] and [5] is based on proportional sharing principle and introducing additional fictitious nodes to remove losses at each line. This method is very popular and the approach concept is simple. However, by introducing the fictitious nodes, the system become larger and requires inverting a sparse matrix that equal to the number of busses in the system plus the additional fictitious nodes that have been introduced.

Gubina *et al.* [6] proposed a method that uses nodal generation distribution factor (NGDF) to trace the active and reactive power. This method uses searching algorithm and applies the proportional sharing principle in the networks. The problem of this method is it does not approach the contribution of generators to line losses. Kirschen *et al.* [7] is based on the concept of generator domains, commons and links. These network characteristics need to be defined first and then the share or a generator or load to a line can be obtained. The disadvantage of this method is that the contribution of each generator in each common is assumed to be the same.

The power flow tracing that uses basic circuit theories method is proposed in [8]. The use of superposition theory, equivalent current injection and equivalent impedance is the base for this method. However, this method is never considered the effect of injected MVAR. Thus this method can be improved. Reference [9] proposed the power flow tracing technique by introducing dominions contribution to the active and reactive power flows. It is a lower-level algorithm in which the concepts of source dominions and common branches are used in [4]-[5], as opposed to commons and links used in [7]. However, this method never applies for tracing the generators' shares to losses.

Modified topological generation and load distribution factors are proposed in [10]. This method introduced decoupled power flow to overcome the losses problem. This method also introduces equivalent model of a line for reactive power tracing. Regarding this method, the effects of line charging to the original generators and loads are integrated. However, the actual contributions from individual generators to lines and loads have been ignored.

The above mentioned disadvantages have been the reason to develop an improved method, (PTM) that can tackles the contribution of individual generator to particular line, loss and load. The method is based on proportional tree method (PTM) proposed in [2]-[3] and adapt the idea of equivalent model of a line [10] with some improvement. The concept is presented in the next

* School of Electrical System Engineering, Universiti Malaysia Perlis (UniMAP) 01000 Kangar, Perlis, Malaysia.

+ Faculty of Electrical Engineering, Universiti Teknologi Malaysia, Johor 81310, Malaysia.

Faculty of Electrical and Electronic, Universiti Malaysia Pahang, 26300, Kuantan Pahang, Malaysia.

^ School of Computer and Communication Engineering, Universiti Malaysia Perlis (UniMAP) 01000 Kangar, Perlis, Malaysia.

1 Corresponding author;
Tel: + 60-4-9851608, Fax: + 60-4-9861431.
E-mail: mherwan@unimap.edu.my

section.

2. BASIC CONCEPT

This paper uses the convention proposed in [2] and [3] with some modifications. It uses proportional sharing principle proposed in [4] and [5] and assumed the node or bus in the system as a perfect mixer. Proportional sharing principle looks at the node or bus where the power inflows are equal to power outflows. This principle is applied for every power outflow at the node in the system and the coefficient at each node is obtained. Then, all buses in the system are rearranged conceptually like a tree. In [2] and [3], PTM is used to trace the transmission cost allocation. The same convention is used with some modifications to develop this power flow and loss tracing algorithm. By applying the PTM, the losses at each transmission lines can be traced and attributed to which generator.

3. EQUIVALENT TRANSMISSION MODEL

Before proceed to the concept of PTM for reactive power flow tracing, the equivalent π model of a line is introduced. Although the transmission losses of reactive power depend on line charging, it is also possible to displace the reactive powers G_{Qi} and G_{Qj} produced by shunt admittances $B_{sh/2,ij}$ into nearby buses as follow [10]:

$$G_{Qi} = V_i^2 B_{sh/2,ij} \tag{1}$$

$$G_{Qj} = V_j^2 B_{sh/2,ij} \tag{2}$$

where V_i is the voltage at sending end and V_j is the voltage at the receiving end of the line. Figure 1 shows this equivalent model of line *i-j*.

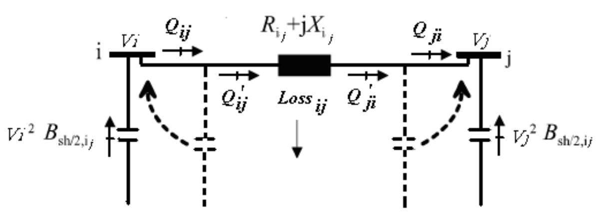

Fig. 1. Equivalent model of the line *i-j* (type 1).

From Figure 1, it can be seen that line *i-j* has the reactive power absorption due to reactance X_{ij} as follows [10]:

$$Loss_{ij} = I_{ij}^2 X_{ij} \tag{3}$$

where I_{ij} is the current through the line *i-j*. To make the system lossless, each of reactive losses is attributed to its sending end. This will follow the concept of proportional sharing principle that proposed in [4] and [5].

However, this equivalent model needs to be considered carefully due to line charging of each transmission line will give some contributions to the reactive power flow in the system. Figures 2, 3 and 4 show the probability of line flow at each line and the assumption that has been made for reactive power flow tracing purpose.

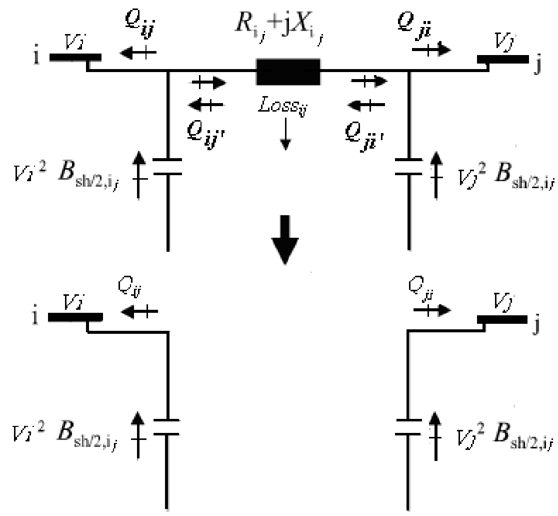

Fig. 2. Equivalent model type 2.

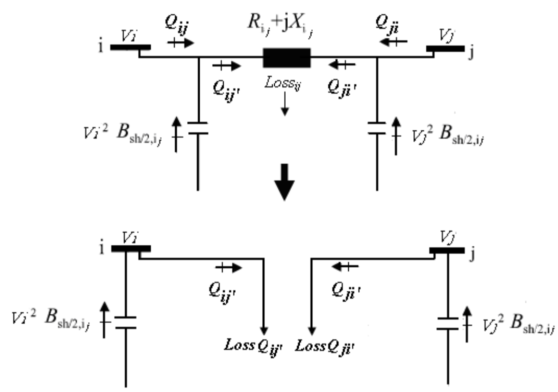

Fig. 3. Equivalent model type 3.

The displacements of shunt admittances are applied if the reactive powers are flows from sending-end to the receiving-end. If the reactive power flows through into both sending and receiving end (type 2), it means that this line is totally supplied by charging megavars. Thus, the contribution of individual generators to this line can be ignored. If the reactive power flows as shown in Figure 3, that particular line will be assumed as load. This is due to the reactive power that flows from both sending and receiving end flowing into the transmission line. Equivalent model type 4 shown in Figure 4 applies displacement of the shunt admittance for sending-end only and treats the reactive power flow as load.

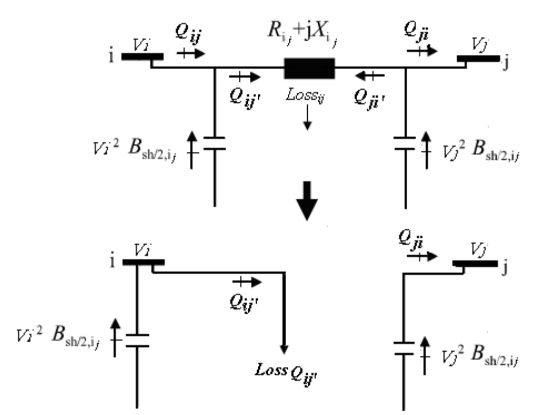

Fig. 4. Equivalent model type 4.

4. PTM CONCEPT

The development process of PTM can be illustrated beginning with small (5-bus test system) power network with AC power flow solutions as shown in Figure 5.

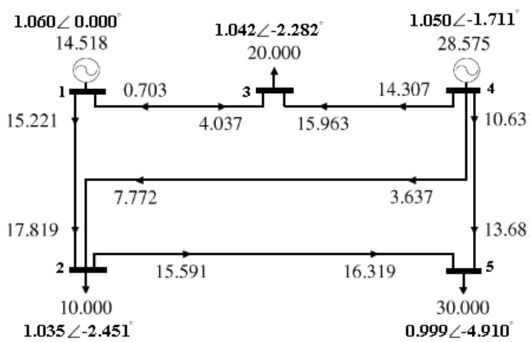

Fig. 5. Five-bus test system with nodal voltages and the reactive power flows in megavolt ampere reactive.

It can be observed from Figure 5 that buses 1 and 4 are the PV bus, while buses 2, 3 and 5 are PQ bus. This system consists of six transmission lines connected to each other. This test system can be obtained in [1] and [10].

Figure 6 shows the test system after introducing the equivalent π model of line. It can be seen that only line 1-3 is not applied for the displacements of reactive powers by shunt admittance due to this line is equal to model type 2. It also can be seen that the integration of the generators with the reactive powers by shunt admittance and the contribution of charging megavars to the loads. The integration of generator, Gi_{int} for each generator bus can be obtained using Equation 4 as follows [10]:

$$Gi_{\text{int}} = Gi + \sum_{n \in Qsh} G_{Qk,n} \tag{4}$$

where Gi, G_{Qk} and Qsh are original generator at bus i, displacement reactive power produced by shunt admittance and number of shunt admittance at bus i respectively.

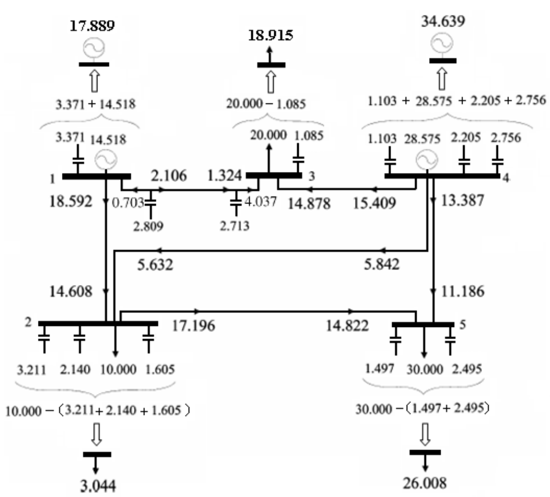

Fig. 6. Reactive power flows in megavolt ampere reactive after introducing the equivalent model line.

Figure 7 shows the lossless system of this test system. It can be seen that the loss at each transmission line have been attributed to the sending end of each line and treated as additional load. Note that line 1-3 is ignored because of no contribution of any generator in the system to this line as discussed before.

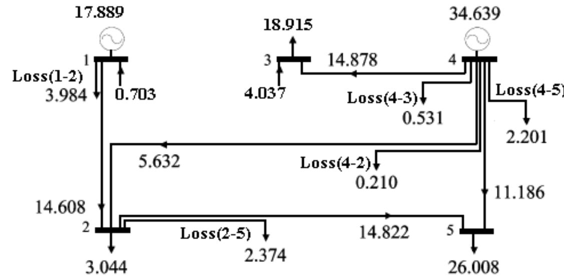

Fig. 7. Lossless system with attributed losses to the sending end of each line.

After lossless system is obtained, the concept of PTM can be applied at each bus. Conceptually, the test system is modified and constructed like a tree. The arrangement of the buses to obtain the reactive power outflow (Qo_{i-j}) coefficients of each line is shown in Figure 8.

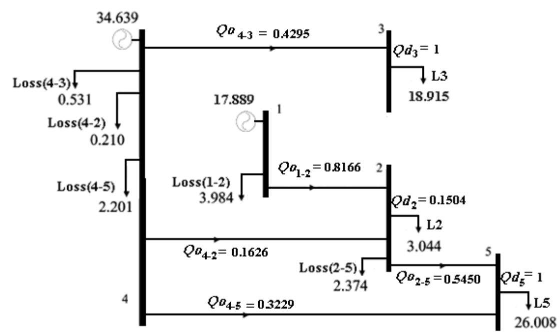

Fig. 8. Model of test system after applied PTM.

The coefficient of Qo_{i-j} is obtained using proportional sharing principle represented by Equation 5 as follows:

$$Qo_{i-j} = \frac{Q_{i-j}}{Q_i} \tag{5}$$

where Q_{i-j} is the reactive power flow from bus i to j and Qi is the total through reactive power of bus i. The modification of PTM concept is applied to the loss at each line as discussed before. The loss at coefficient at each line, $QLoss_{i-j}$ is obtained using the Equation 6 as follows:

$$QLoss_{i-j} = \frac{Loss_{i-j}}{Q_i} \tag{6}$$

where $Loss_{i-j}$ is the loss at each line that is attributed to its sending-end bus.

Since the system is lossless, it is easy to trace the contribution of individual generators to lines, losses and loads. Thus, the next subsection will explain about these power tracing algorithms.

Generator Contribution Factor

From the tree model in Figure 8, the contribution factors can be calculated from the generator to a line or load directly. However, by referring to Figure 6, the generator at bus 1 and bus 4 are integrated with the reactive power of shunt admittances. Thus, the contribution factor from new generator *i* to line *i-j* through path *k* ($C^{k}_{Gi\text{int},i-j}$) can be calculated using the following expression:

$$C^{k}_{Gi\,\text{int},i-j} = \prod_{n\in Nl} Qo_{i-j,n} \tag{7}$$

where *Nl* is the number of line to reach line *i-j*.

To obtain the total contribution factor from integrated generator *i* to line *i-j*, $C^{Gi\text{int}}_{i-j}$ through all related paths, the expression below is used:

$$C^{Gi\,\text{int}}_{i-j} = \sum_{k\in Np} C^{k}_{Gi\,\text{int},i-j} \tag{8}$$

where *Np* is the total number of paths that connecting between generator *i* to line *i-j*.

Generators' Contribution to Line Flow

Generator's contribution at each line, $Q^{Gi\text{int}}_{i-j}$ can be calculated based on contribution factors that obtained earlier. It can be obtained by using the following equation:

$$Q^{Gi\,\text{int}}_{i-j} = \frac{C^{Gi\,\text{int}}_{i-j} \times Gi\,\text{int} \times Q_{i-j}}{\sum_{k=1}^{Ng} C^{Gi\,\text{int}}_{i-j} \times Gk\,\text{int}} \tag{9}$$

where *Gk*int is the integrated reactive power from generator *k* and *Ng* is the number of generator bus. To obtain the contribution of original reactive generator to each line, Equation 10 is used as follows:

$$Q^{Gi}_{i-j} = \frac{Gi \times Q^{Gi\,\text{int}}_{i-j}}{Gi\,\text{int}} \tag{10}$$

Generator to Line Loss Contribution Factor

After obtaining the integrated generator's contribution to all lines, the contribution factor to the losses at each line can be calculated by the same method that applied to the line flow above. The contribution factor of integrated generator *i* to the loss at each line, $Loss_{i-j}$ through path *k* ($C^{k}_{Gi\text{int}},Loss_{i-j}$) can be obtained by applying Equation 11 below:

$$C^{k}_{Gi\,\text{int}} Loss_{i-j} = \Big(\prod_{n\in Nl-1} Qo_{i-j,n}\Big) \times QLoss_{i-j} \tag{11}$$

where *Nl-1* is the number of line to reach the loss at each line (coefficient of line *i-j* is not included), $Qo_{i-j,n}$ is obtained using Equation 5 and $QLoss_{i-j}$ is obtained using Equation 6.

To obtain the total contribution factor from integrated generator *i* to $Loss_{i-j}$, $C^{Gi\text{int}},Loss_{i-j}$ through all related paths, the expression below is used:

$$C^{Gi\,\text{int}} Loss_{i-j} = \sum_{k\in Np} C^{k}_{Gi\,\text{int},} Loss_{i-j} \tag{12}$$

where *Np* is the total number of paths that connecting the generator *i* to $Loss_{i-j}$.

Generators' Contribution to Line Loss

Generator's contribution to line loss, $Q^{Gi\text{int},} Loss_{i-j}$ can be calculated based on contribution factors that obtained earlier. It can be obtained by using the following equation:

$$QLoss^{Gi\,\text{int}}_{i-j} = \frac{C^{Gi\,\text{int}} Loss_{i-j} \times Gi\,\text{int} \times Loss_{i-j}}{\sum_{k=1}^{Ng} C^{Gi\,\text{int}} Loss_{i-j} \times Gk\,\text{int}} \tag{13}$$

where *Gk*int is the integrated reactive power from generator *k* and *Ng* is the number of generator bus. To obtain the contribution of original reactive generator to each line loss, $C^{Gi},Loss_{i-j}$ Equation 14 is used as follows:

$$QLoss^{Gi}_{i-j} = \frac{Gi \times QLoss^{Gi\,\text{int}}_{i-j}}{Gi\,\text{int}} \tag{14}$$

Generator to Load Contribution Factor

Since the method is based on proportional sharing principle, the load is treated as one of the power outflows from the node, the same principle that applied to the loss that obtained earlier. Thus, to trace the contribution of generator to the load, Equation 15 can be used to obtain additional power outflow coefficient:

$$Qd_i = \frac{L_i}{Q_i} \tag{15}$$

where Qd_i is the power outflow coefficient to load *i*, L_i is load at bus *i* and *Qi* is the total power through bus *i*.

By referring to Figure 8 again, the contribution factor from generator to load can be traced directly. The contribution factor from integrated generator *i* to load L_j through path *k* ($C^{k}_{Gi\text{int},Lj}$) can be calculated by using Equation 16 as follows:

$$C^{k}_{Gi\,\text{int},L_j} = \Big(\prod_{n\in Nl} Qo_{i-j,n}\Big) \times Qd_j \tag{16}$$

where *Nl* is number of lines to reach load *j*. From Equation 16, it can be seen that the contribution of particular load (Qd_j) on the overall contribution factors. Total contribution factor from integrated generator *i* to load $C^{Gi\text{int}}_{Lj}$ through all related paths including outflow coefficient of load *j* can be obtained by using the expression:

$$C^{Gi\,\text{int}}_{Lj} = \sum_{k\in Np} C^{k}_{Gi\,\text{int},Lj} \tag{17}$$

where *Np* is the total number of paths that connect the generator *i* to load *j*.

Generators' Contribution to Load

Integrated generator's contribution at each load, $Q^{Gi\text{int}}_{Lj}$ can be calculated based on contribution factors by rearranging Equations 16 and 17. It yields the following equation:

$$Q_{Lj}^{Gi\,\mathrm{int}} = \frac{[C_{Lj}^{Gi\,\mathrm{int}} \times Gi\,\mathrm{int}] \times L_j}{\sum_{k=1}^{Ng} C_{Lj}^{Gk\,\mathrm{int}} \times Gk\,\mathrm{int}} \tag{18}$$

where Gkint is the integrated reactive power from the generator k and Ng is the number of generator bus. To obtain the contribution of original reactive generator to each load, Q^{Gi}_{Lj} Equation 19 is used as follows:

$$Q_{Lj}^{Gi} = \frac{Gi \times Q_{Lj}^{Gi\,\mathrm{int}}}{Gi\,\mathrm{int}} \tag{19}$$

5. NUMERICAL EXAMPLES AND DISCUSSION

Wang [2] and Xiao [3] specified the PTM for transmission cost allocation. The same convention is followed with some modifications have been made to suit the reactive power flow tracing purpose. In order to verify the feasibility and effectiveness of this modified method, a numerical calculation is performed at line 2-5, where the line is not connected directly to any generator bus. By referring to Figure 8, the contribution factor from integrated G1 and G4 for line 2-5 is calculated as follows:

$C^1_{G1\mathrm{int},2\text{-}5} = Qo_{1\text{-}2} \times Qo_{2\text{-}5} = 0.4450$

$C^1_{G4\mathrm{int},2\text{-}5} = Qo_{4\text{-}2} \times Qo_{2\text{-}5} = 0.0886$

The total contribution factor from integrated G1 and G4 to line 2-5 is:

$C^{G1\mathrm{int}}_{2\text{-}5} = C^1_{G1\mathrm{int},2\text{-}5} = 0.4450$

$C^{G4\mathrm{int}}_{2\text{-}5} = C^1_{G4\mathrm{int},2\text{-}5} = 0.0886$

The contribution of integrated G1 and G4 to this line can be obtained using Equation 9 as:

$$Q_{2-5}^{G1\,\mathrm{int}} = \frac{(0.4450 \times 17.889) \times 14.822}{(0.4450 \times 17.889) + (0.0886 \times 34.639)} = 10.698$$

$$Q_{2-5}^{G2\,\mathrm{int}} = \frac{0.0886 \times 34.639 \times 14.822}{(0.4450 \times 17.889) + (0.0886 \times 34.639)} = 4.124$$

From the above calculation, the total contribution of integrated G1 and G4 is equal to the receiving power flow in line 2-5, which is 14.822 MVar. To obtain the contribution of original G1 and G4 to line 2-5, Equation 10 is used as:

$$Q_{2-5}^{G1} = \frac{14.518 \times 10.698}{17.889} = 8.682\,MVar$$

$$Q_{2-5}^{G4} = \frac{28.575 \times 4.124}{34.639} = 3.402\,MVar$$

To trace the contribution of integrated G1 and G4 to the loss at line *2-5*, $Loss_{2\text{-}5}$ where Gkint is the integrated reactive power from generator k and Ng is the number of generator bus. Equations 11 and 12 are used as follows:

$C^1_{G1\mathrm{int},}Loss_{2\text{-}5} = Qo_{1\text{-}2} \times QLoss_{2\text{-}5} = 0.0713$

$C^1_{G4\mathrm{int},}Loss_{2\text{-}5} = Qo_{4\text{-}2} \times Qloss_{2\text{-}5} = 0.0142$

The total contribution factor from G1 and G4 to $Loss_{2\text{-}5}$ is:

$C^{G1\mathrm{int}}Loss_{2\text{-}5} = C^1_{G1\mathrm{int},}Loss_{2\text{-}5} = 0.0713$

$C^{G4\mathrm{int}}Loss_{2\text{-}5} = C^1_{G4\mathrm{int},}Loss_{2\text{-}5} = 0.0142$

The contribution of integrated G1 and G4 to this line loss can be obtained using Equation 13 as:

$$Q_{2-5}^{G1\,\mathrm{int}} = \frac{(0.0713 \times 17.889) \times 2.374}{(0.0713 \times 17.889) + (0.0142 \times 34.639)} = 1.713$$

$$Q_{2-5}^{G2\,\mathrm{int}} = \frac{0.0142 \times 34.639 \times 2.374}{(0.0713 \times 17.889) + (0.0142 \times 34.639)} = 0.661$$

It can be seen that the total of integrated G1 and G4 is equal to the reactive loss of line 2-5, which is 2.374 MVar. However, to obtain the original generator to this loss, Equation 14 is used and the result as follows:

$$Q_{Loss,2-5}^{G1} = \frac{14.518 \times 1.713}{17.889} = 1.390\,MVar$$

$$Q_{Loss,2-5}^{G4} = \frac{28.575 \times 0.661}{34.639} = 0.545\,MVar$$

From the calculation above, it can be seen that the total contribution of original generator G1 and G4 is equal to 1.9367 Mvar. Thus, 0.439 Mvar is contributed by charging megavars of this line.

At this point, the contribution of each generator to line 2-5 and the losses contributed by each generator has been calculated. Table 1 shows the result of power contribution from the individual generators to line flows and losses.

Table 1 shows the contribution of generators bus 1 and bus 4 to the lines and the line losses caused by each integrated generator and original generator. It can be seen that line 1-2 has no contribution from generator bus 4, while for line 4-2, 4-3 and 4-5, there are no contribution from generator bus 1. From this table also it can be seen that no contribution from both generators to line 1-3 due to said that this line is totally supplied from line charging megavars of this line. The comparison with the method proposed in [8] shows big different results. It can be seen some of the results has negative values. This means that the current flow is backward for that particular generator at that line. It also shows that the total reactive power flow is not equal with the power flow solution (Figure 5) due to effects of shunt admittance at each line when calculating the current flow tracing [8].

To calculate the actual generators' contribution to the loads, the same method is used as above. Again, referring Figure 8, the contribution factor from integrated G1 and G4 for load 5 is calculated as follows:

$\mathrm{C}^1_{\mathrm{G1int,L5}} = \mathrm{Qo}_{1\text{-}2} \times \mathrm{Qo}_{2\text{-}5} \times \mathrm{Qd}_5 = 0.4450$

$\mathrm{C}^1_{\mathrm{G4int,L5}} = \mathrm{Qo}_{4\text{-}2} \times \mathrm{Qo}_{2\text{-}5} \times \mathrm{Qd}_5 = 0.0886$

$C^2_{G4\mathrm{int},L5} = Qo_{4\text{-}5} \times Qd_5 = 0.3229$

The total contribution factor from integrated G1 and G4 to load 5 is:

$C^{G1\mathrm{int}}_{L5} = C^1_{G1\mathrm{int},2\text{-}5} = 0.4450$

$C^{G4\mathrm{int}}_{L5} = C^1_{G4\mathrm{int},2\text{-}5} + C^2_{G4\mathrm{int},2\text{-}5} = 0.4115$

Table 1. Reactive power contribution from individual generators to line flows and losses in megavolt ampere reactive (Mvar) for 5-bus system.

Line ID	Line power supplied by		Total Power Flow	Line loss caused by		Total Loss	Line power supplied by original		Line loss caused by original		Line power supplied by (results from [8])		Total Power Flow
	G1int	G4int		G1int	G4int		G1	G4	G1	G4	G1	G4	
Line 1-2	14.605	0.000	14.605	3.987	0.000	3.987	11.853	0.000	3.235	0.000	12.664	5.851	18.515
Line 1-3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.205	-3.149	2.056
Line 4-2	0.000	5.630	5.630	0.000	0.213	0.213	0.000	4.644	0.000	0.175	-2.037	7.848	5.811
Line 2-5	10.698	4.124	14.822	1.715	0.661	2.376	8.683	3.402	1.392	0.545	9.117	8.039	17.156
Line 4-3	0.000	14.872	14.872	0.000	0.532	0.532	0.000	12.273	0.000	0.439	2.016	13.388	15.404
Line 4-5	0.000	11.186	11.186	0.000	2.201	2.201	0.000	9.228	0.000	1.816	3.285	9.974	13.259

The contribution of integrated G1 and G4 to this line can be obtained using Equation 17 as:

$$Q_{L5}^{G1\text{int}} = \frac{(0.445\times17.889)\times26.008}{(0.445\times17.889)+(0.4115\times34.639)} = 9.32MVar$$

$$Q_{2L5}^{G4\text{int}} = \frac{0.4115\times34.639\times26.008}{(0.445\times17.889)+(0.4115\times34.639)} = 16.688MVar$$

Finally, to obtain the actual contribution of G1 and G4 to load 5, Equation 18 is used as follows:

$$Q_{L5}^{G1} = \frac{14.518\times9.32}{17.889} = 7.563MVar$$

$$Q_{L5}^{G4} = \frac{28.575\times16.688}{34.639} = 13.767MVar$$

The contribution of the generators to the Load 5 is shown in Figure 9 where the comparison of PTM with the method as proposed in [8] again is done.

It can be seen that the total contribution for PTM for Load 5 is equal to 26.008 MVar while for method proposed in [8], the total contribution is equal to 30 MVar. Discrepancies are expected due to effect of line charging megavars that taken into account in PTM. By referring back to Figure 6, the demand in bus 5 is changed to 26.008 due to line charging megavars that introduced in equivalent model of a line. However, it can also be seen that generator 4 contributes more to Load 5 compared to generator 1 for both methods. This is due to the location of Load 5 being nearer and connected directly to generator bus 4.

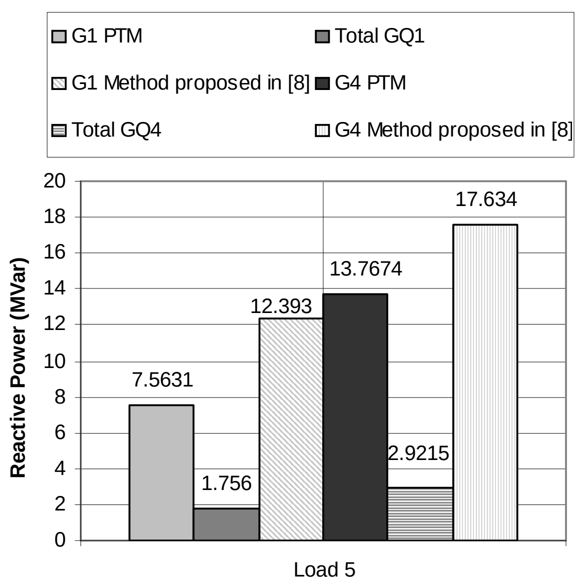

Fig. 9. Reactive power contribution by each generator to load 5 in MVar.

Basically, several testing system have been carried out to see the veracity and feasibility of the proposed method. The method is developed in Matlab. Table 2 shows the integrated generators' contribution for reactive line flows and losses for IEEE-14 bus system. It can be seen that there is no contribution from generator bus 1. This is because from power flow solution, generator bus 1 has negative value and it means that this generator did not supply any reactive power into the system. This table also shows that reactive power flow at lines 1-5, 2-3 and 4-2 are zero. This is because of equivalent π model that applied to these lines. Lines 1-5 and 2-3 are equivalent to model type 3 while for line 4-2 is equivalent to model type 4 that have been discussed at early section of this paper. Thus the reactive power flows for these lines are treated as losses.

Table 3 shows the results of original generators' contribution to line flows and losses for IEEE-14 bus system. This result is obtained using Equations 10 and 14.

Table 2. Reactive power contribution from integrated generators to line flows and losses in megavolt ampere reactive (MVar) for IEEE-14 Bus test system.

Line ID	Line power supplied by					Total power flow	Line loss caused by					Total loss
	G1 int	G2 int	G3 int	G6 int	G8 int		G1 int	G2 int	G3 int	G6 int	G8 int	
2-1	0.000	17.434	0.000	0.000	0.000	17.434	0.000	13.129	0.000	0.000	0.000	13.129
1-5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.755	1.401	0.000	3.248	11.403
2-3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.952	3.834	0.000	0.000	9.786
4-2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.440	0.000	3.342	4.782
2-5	0.000	0.300	0.000	0.000	0.000	0.300	0.000	2.759	0.000	0.000	0.000	2.759
3-4	0.000	0.000	4.177	0.000	0.000	4.177	0.000	0.000	0.946	0.000	0.000	0.946
8-7	0.000	0.000	0.000	0.000	17.160	17.160	0.000	0.000	0.000	0.000	0.460	0.460
7-4	0.000	0.000	0.000	0.000	9.680	9.680	0.000	0.000	0.000	0.000	1.700	1.700
4-5	0.000	0.000	4.280	0.000	9.920	14.200	0.000	0.000	0.488	0.000	1.132	1.620
5-6	0.000	0.238	2.355	0.000	5.457	8.050	0.000	0.131	1.293	0.000	2.996	4.420
6-11	0.000	0.025	0.250	2.587	0.578	3.440	0.000	0.001	0.009	0.090	0.020	0.120
6-12	0.000	0.017	0.171	1.767	0.395	2.350	0.000	0.001	0.011	0.113	0.025	0.150
6-13	0.000	0.050	0.493	5.113	1.143	6.800	0.000	0.003	0.031	0.316	0.071	0.420
7-9	0.000	0.000	0.000	0.000	4.980	4.980	0.000	0.000	0.000	0.000	0.800	0.800
9-4	0.000	0.000	0.000	0.000	0.430	0.430	0.000	0.000	0.000	0.000	1.300	1.300
9-10	0.000	0.000	0.000	0.000	4.180	4.180	0.000	0.000	0.000	0.000	0.040	0.040
9-14	0.000	0.000	0.000	0.000	3.360	3.360	0.000	0.000	0.000	0.000	0.250	0.250
11-10	0.000	0.012	0.118	1.218	0.272	1.620	0.000	0.000	0.002	0.015	0.003	0.020
12-13	0.000	0.006	0.054	0.564	0.126	0.750	0.000	0.000	0.000	0.000	0.000	0.000
13-14	0.000	0.012	0.119	1.233	0.276	1.640	0.000	0.001	0.008	0.083	0.019	0.110

Table 3. Reactive power contribution from original generators to line flows and losses in megavolt ampere reactive (MVar) for IEEE-14 Bus test system.

Line ID	Line power supplied by original					Line loss caused by				
	G1	G2	G3	G6	G8	G1	G2	G3	G6	G8
2-1	0.000	14.972	0.000	0.000	0.000	0.000	10.941	0.000	0.000	0.000
1-5	0.000	0.000	0.000	0.000	0.000	0.000	5.801	1.256	0.000	3.248
2-3	0.000	0.000	0.000	0.000	0.000	0.000	5.111	3.438	0.000	0.000
4-2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.291	0.000	3.342
2-5	0.000	0.258	0.000	0.000	0.000	0.000	2.299	0.000	0.000	0.000
3-4	0.000	0.000	3.746	0.000	0.000	0.000	0.000	0.848	0.000	0.000
8-7	0.000	0.000	0.000	0.000	17.160	0.000	0.000	0.000	0.000	0.460
7-4	0.000	0.000	0.000	0.000	9.680	0.000	0.000	0.000	0.000	1.700
4-5	0.000	0.000	3.838	0.000	9.920	0.000	0.000	0.438	0.000	1.132
5-6	0.000	0.205	2.112	0.000	5.457	0.000	0.109	1.159	0.000	2.996
6-11	0.000	0.022	0.224	2.587	0.578	0.000	0.001	0.008	0.055	0.020
6-12	0.000	0.015	0.153	1.767	0.395	0.000	0.001	0.010	0.069	0.025
6-13	0.000	0.043	0.442	5.113	1.143	0.000	0.003	0.027	0.194	0.071
7-9	0.000	0.000	0.000	0.000	4.980	0.000	0.000	0.000	0.000	0.800
9-4	0.000	0.000	0.000	0.000	0.430	0.000	0.000	0.000	0.000	1.300
9-10	0.000	0.000	0.000	0.000	4.180	0.000	0.000	0.000	0.000	0.040
9-14	0.000	0.000	0.000	0.000	3.360	0.000	0.000	0.000	0.000	0.250
11-10	0.000	0.010	0.105	1.218	0.272	0.000	0.000	0.001	0.009	0.003
12-13	0.000	0.005	0.049	0.564	0.126	0.000	0.000	0.000	0.000	0.000
13-14	0.000	0.010	0.107	1.233	0.276	0.000	0.001	0.007	0.051	0.019

6. CONCLUSION

This paper has presented a method for calculating the contribution from individual generators to reactive line flows, transmission losses and loads. The method uses a convention proposed in [2], [3] and [10] with some modifications. In addition, the equivalent transmission models are introduced and have been applied in this tracing method. The method is simple and accurate. Accordingly, illustrative network is selected as test case to show the feasibility and veracity of the method.

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APPENDIX

Table A. Bus data of IEEE-14 Bus system.

	Voltage		Generation		Load		Injected
Bus No.	Mag(p.u)	Ang(deg)	P(MW)	Q(MVar)	P(MW)	Q(MVar)	Mvar
1	1.06	0	232.39	-16.55	0	0	0
2	1.045	-4.983	40	43.56	21.7	12.7	0
3	1.01	-12.725	0	25.08	94.2	19	0
4	1.018	-10.313	0	0	47.8	-3.9	0
5	1.02	-8.774	0	0	7.6	1.6	0
6	1.07	-14.221	0	12.73	11.2	7.5	0
7	1.062	-13.36	0	0	0	0	0
8	1.09	-13.36	0	17.62	0	0	0
9	1.056	-14.939	0	0	29.5	16.6	21.2
10	1.051	-15.097	0	0	9	5.8	0
11	1.057	-14.791	0	0	3.5	1.8	0
12	1.055	-15.076	0	0	6.1	1.6	0
13	1.05	-15.156	0	0	13.5	5.8	0
14	1.036	-16.034	0	0	14.9	5	0
		Total:	272.39	82.44	259	73.5	21.2

Table B. Line data of IEEE-14 Bus system.

Line No	From Bus	To Bus	From Bus Injection		To Bus Injection		Loss	
			P (MW)	Q (Mvar)	P (MW)	Q (Mvar)	P (MW)	Q (Mvar)
1	1	2	156.88	-20.4	-152.59	27.68	4.298	13.12
2	1	5	75.51	3.85	-72.75	2.23	2.763	11.41
3	2	3	73.24	3.56	-70.91	1.6	2.323	9.79
4	2	4	56.13	-1.55	-54.45	3.02	1.677	5.09
5	2	5	41.52	1.17	-40.61	-2.1	0.904	2.76
6	4	3	23.66	-4.84	-23.29	4.47	0.373	0.95
7	8	7	0	17.62	0	-17.16	0	0.46
8	4	7	28.07	-9.68	-28.07	11.38	0	1.7
9	5	4	61.67	-14.2	-61.16	15.82	0.514	1.62
10	5	6	44.09	12.47	-44.09	-8.05	0	4.42
11	6	11	7.35	3.56	-7.3	-3.44	0.055	0.12
12	6	12	7.79	2.5	-7.71	-2.35	0.072	0.15
13	6	13	17.75	7.22	-17.54	-6.8	0.212	0.42
14	7	9	28.07	5.78	-28.07	-4.98	0	0.8
15	4	9	16.08	-0.43	-16.08	1.73	0	1.3
16	9	10	5.23	4.22	-5.21	-4.18	0.013	0.03
17	9	14	9.43	3.61	-9.31	-3.36	0.116	0.25
18	11	10	3.8	1.64	-3.79	-1.62	0.013	0.03
19	12	13	1.61	0.75	-1.61	-0.75	0.006	0.01
20	13	14	5.64	1.75	-5.59	-1.64	0.054	0.11