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Determination of Power Flow and Loss Allocation Using Superposition Theory Method

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Abstract – *This paper discussed a method to determine power flow and loss from individual generator to particular load and line flows for transmission open access. Based on solved load flow, the method uses basic circuit theories including equivalent impedance, equivalent current injection and superposition theory as a foundation of algorithm's development. IEEE 14-bus test system and 4-bus test system were used to illustrate the veracity and effectiveness of the method. Comparison of the results with previous methods is also given.*

Keywords – Deregulation, load flow study, superposition law, transmission open access.

1. INTRODUCTION

With the deregulation of electric power systems, the determination of power flow and loss in the meshed network has become very important in the interest of fair competition. Furthermore, it is interesting to know, which generator supply the power to particular line and how much its contribution to the loads. Since the nonlinear nature of power flow, it is difficult to trace the power flow in the network accurately. Thus the studies of this area are enhanced and developed year by year to give the opportunity to the researchers to come out the algorithms for power and loss tracing method.

There are many algorithms that have been proposed which can allocate the contributions of power and loss from particular generator through the transmission network to the lines and loads [1]-[7], [12]-[14]. Reference [1] proposed a procedure for allocating transmission losses to generators and loads based on the network Zbus matrix. This method is needed an attention to the fact that the values of Zbus are very sensitive to small changes on network parameters. In [3], incremental loss transmission allocation under pool dispatch is discussed. From this discussion, it is shown that the incremental loss allocation among bus power injections is arbitrary and therefore cannot be used to allocate losses in nondiscriminatory manner. Thus, some mathematical formulas are developed to obtain unique incremental loss allocation that makes possible for equivalent power exchanges between generators and loads. In [4], the modified topological generation and load distribution factors (TGDF/TLDF) method for the power flow tracing is reported. The proposed approach introduces several novelties of which different consideration of the transmission losses is the most important since it enables decoupling of extended matrices.

The method based on network topology and proportional sharing assumption was proposed in [5]. In the method, the network characteristics including common, links and state of graph need to be define first and then the contribution of a load or generator to a line can be obtained. In [12], three bus-oriented schemes based on generalized generation distribution factors (GGDFs) and generalized load distribution factors (GLDFs) are proposed to allocate the transmission losses to market participants. This bus-oriented method is aimed to reducing the distribution factor computation and reflecting the activity in a competitive market. The method based on the physical-flow-based approach to allocate transmission loss was proposed in [6].

J. Bialek proposed a method based on topological generation, load distribution factors and proportional sharing assumption and use the upstream and downstream-looking algorithm to trace the flows [7], [13]. In [14], an improve usage allocation method is reported. The method use proportional sharing principle and graph theory to trace the relationship between current sources and current sinks.

Basically, the algorithm's development in this paper is referred to [2]. In [2], the concerned of the author is regarding singular characteristic of full admittance matrix that resulted of additional formulas to overcome singular problem. From [2] also, the equations is reviewed and improved.

2. CONCEPT OF SUPERPOSITION THEORY METHOD

Basic Concept

Superposition theory method is developed based on basic circuit theories including KCL, KVL and Superposition theory [2]. From [2], equations were reviewed and reduced to meet the requirement of power tracing algorithm. An equation is improved by introducing the effect of injected MVAR that will be discussed later. After power flow solution is obtained, we can identify voltages magnitudes, angles, and total real and reactive power for each bus in the network. This method assumes that the generator bus can be treated as equivalent current

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injection and load bus as equivalent impedance. The apparent power of generator bus *n* and its corresponding current injection can be expressed as [2]:

$$S_{n,G} = \left(P_{n,G} + jQ_{n,G}\right) \tag{1}$$

$$I_{n,G} = \left(\frac{P_{n,G} + jQ_{n,G}}{V_{n,G}}\right) \tag{2}$$

Where n is the number of generator, $V_{n,G}$ is the generator bus voltage, $P_{n,G}$ is the real power and $Q_{n,G}$ is the reactive power for the generator bus. Those elements can be obtained from load flow study.

In power system, generators and loads are not the only sources and/or sinks of complex power. Sytatic Var Compensators (SVCs), shunt capacitors/reactors and line charging capacitances play a vital role in transferring power between generators and loads. Thus, this paper will take this effect and added this effect into corresponding equivalent impedance that derived from [2]. Normal practical in power system, at certain load bus is added shunt capacitors or SVCs to improve or increase the voltage stability. Thus, by integrated this effect, we can derive the corresponding equivalent impedance *($Z_{i,L}$)* at load bus *i* as:

$$Z_{i,L} = \frac{V_{i,L}}{I_{i,L}} = \frac{\left|V_{i,L}\right|^2}{P_{i,L} - j\left(Q_{i,L} - Q_c\right)} \tag{3}$$

Where $V_{i,L}$, $I_{i,L}$ and $S_{i,L} = [P_{i,L} - j(Q_{i,L} - Q_c)]$ are the voltage, current and apparent power of load bus *i* including effect of injected MVAR that obtained from the converged load flow solution respectively. Note that Qc in this equation is the effect of injected MVAR (shunt capacitor or SVCs) at load bus. After the equivalent impedance was integrated into the admittance matrix, the relation between bus voltages and bus current injection can be expressed as [2]:

$$V_{BUS} = Z_{MATRIX} I_G \tag{4}$$

Where $V_{BUS,}$, I_G and Z_{MATRIX} are the bus voltage vector, current injection vector and impedance matrix including the effect of the equivalent impedance respectively. The effect of slack bus is also included in this equation.

Tracing the Voltage at Bus from Each Generator

To trace the voltage, we use superposition law as the foundation of this method development. Only one generator is connected to the system and at the same time the other generators in the system are open circuit. By taking the generators into account one by one, we can express the voltage contribution of each generator to each bus as [2]:

$$\begin{bmatrix} \Delta v_1^n \\ \vdots \\ \Delta v_n^n \\ \vdots \\ \Delta v_N^n \end{bmatrix} = \begin{bmatrix} z_{11} & \cdots & z_{1n} & \cdots & z_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ z_{n1} & \cdots & z_{nn} & \cdots & z_{nN} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ z_{N1} & \cdots & z_{Nn} & \cdots & z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ I_{n,G} \\ \vdots \\ 0 \end{bmatrix} \tag{5}$$

From the expression above, voltage at bus *i* contributed by generator bus *n* *(Δv_i^n)* and the voltage of bus *i* contributed by all generator buses can be written as [2]:

$$\Delta v_i^n = z_{in} * I_{n,G} \tag{6}$$

$$V_i = \sum_{n=1}^{N_G} \Delta v_i^n \tag{7}$$

From these equations, it can be seen that the voltage contributions of each generator to each bus can be calculated accordingly. This information is very important to calculate the power flow and loss allocation.

Tracing the Current Through Each Line

Based on circuit theory concept, the current flow at each line in deregulated network can be determined by [2]:

$$\Delta i_{ij}^n = \left(\Delta v_i^n - \Delta v_j^n\right)\left(g_{ij} + jb_{ij}\right) + \left(jc/2\right)\left(\Delta v_i^n\right) \tag{8}$$

$$\Delta i_{ji}^n = \left(\Delta v_j^n - \Delta v_i^n\right)\left(g_{ij} + jb_{ij}\right) + \left(jc/2\right)\left(\Delta v_j^n\right) \tag{9}$$

Where $y_{ij} = (g_{ij} + jg_{ij})$ is the line admittance from bus *i* to *j* and *c/2* is the line charging susceptance. Δi^n_{ij} and Δi^n_{ji} are the line currents, produced by generator bus *n*, from bus *i* to bus *j* and from bus *j* to bus *i*, respectively.

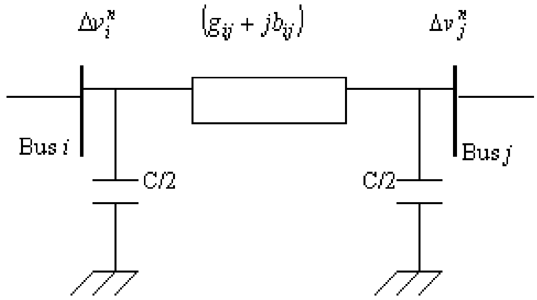

Fig. 1. A Transmission line section model

Table 1. Converged bus solution of 4-bus test system

Bus No.	P (MW)	Q (MVAR)	V (p·u)	Β (°)	Bus Type
1	38.2870	11.1880	1.0500	0.0000	Slack
2	-54.995	-12.9900	1.0071	-4.5582	PQ
3	-30.001	-18.008	1.0195	-3.0850	PQ
4	50.000	28.7550	1.0700	1.0147	PV

Table 2. Line parameter data

Line No.	From Bus	To Bus	R (p·u)	X (p·u)
1	1	2	0.08	0.40
2	1	3	0.12	0.50
3	3	2	0.10	0.40
4	4	2	0.10	0.50
5	4	3	0.00	0.30

Tracing Power Flow and Loss

Since the voltage and current at each bus has been identified, the power flow at every line and the loss in the system can be calculated. The power flow from bus *ii* to bus *j* and the loss produced by generator *n* at each line can be expressed as [2]:

$$\Delta s_{ij}^{n} = V_i \left(\Delta i_{ij}^{n}\right)^* \tag{10}$$

$$P_{ij,Loss}^{n} = \mathrm{Re}\left(\Delta s_{ij}^{n}\right) + \mathrm{Re}\left(\Delta s_{ji}^{n}\right) \tag{11}$$

The power from generator to a load also can be calculated by the same procedure, which is:

$$\Delta i_{i,L}^{n} = \frac{\Delta v_i^n}{Z_{i,L}} \tag{12}$$

Where $\Delta i^{n}_{i,L}$ is the current injection of a load bus contributed by generator bus *n*. Therefore, the power of load bus *i* contributed by generator bus *n* can be written as:

$$\Delta s_{i,L}^{n} = V_i \left(\Delta i_{i,L}^{n}\right)^* \tag{13}$$

The correctness of this method can be verified by comparing the results obtained by expressions above with the converged power solution.

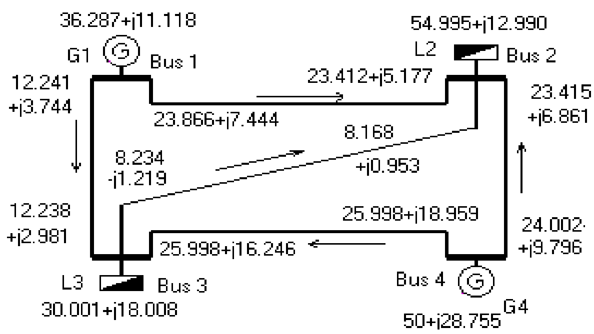

Fig. 2. The power flow solution of the 4-bus test system

3. NUMERICAL EXAMPLES AND DISCUSSION

A number of simulations have been carried out to demonstrate the validity and veracity of the method. The method was implemented by Matlab 6.5 programming language. A load flow program that developed by [8] is used to obtain the system status. The sizes of the test systems are not large, but it is adequate to illustrate the correctness of the method.

Tables 1 and 2 show the converged bus solution and the line parameters of 4-bus test system. This test system is obtained from [2]. Figure 2 shows the network topology of the 4-bus system. There are two generators which are at bus 1 and bus 4 and two loads at bus 2 and bus 3 for this system. The status including the power injections and power flows at both ends of each line are also in Figure 2.

Figure 3 shows the equivalent current injections of bus 1 and bus 4, and the voltages contributed by each generator. From here, it can be seen that the sum of bus voltages contributed by each generator is equal to the converged bus voltages.

Figures 4 and 5 show the line currents and powers contributed by each generator, respectively. Note that only the values as indicated by arrows are shown. From Figure 5, it can be seen that the KCL of each bus and KVL of each loop are satisfied. The fulfillment of KCL and KVL are both for each individual generator and the full system.

Figure 6 shows the losses contributed by each generator. It can be seen that the total line losses contributed by generator bus 1 and bus 4 are 0.717 MW and 0.575 MW, respectively. The sum of line losses produced by each generator is the same as the line losses by load flow program.

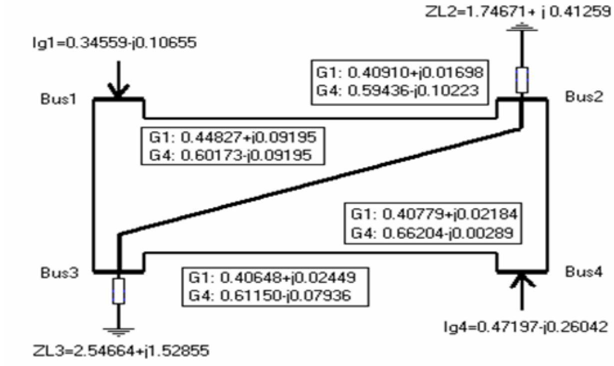

Fig. 3. Voltage tracing result of the 4-bus test system

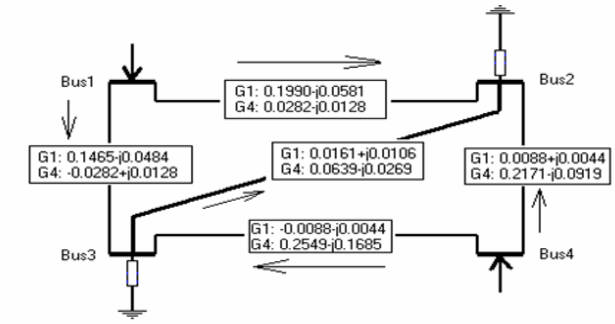

Fig. 4. Current tracing result of 4-bus test system

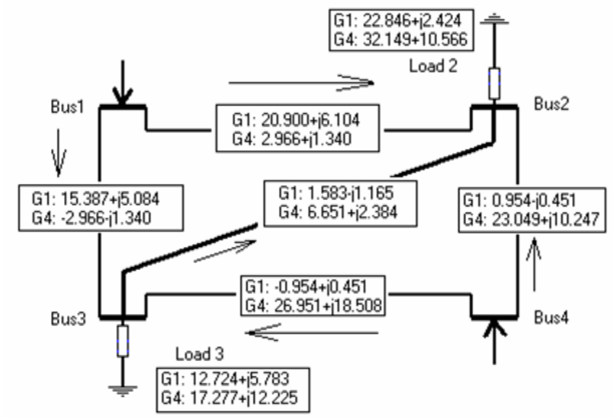

Fig. 5. Power tracing result of 4-bus test system

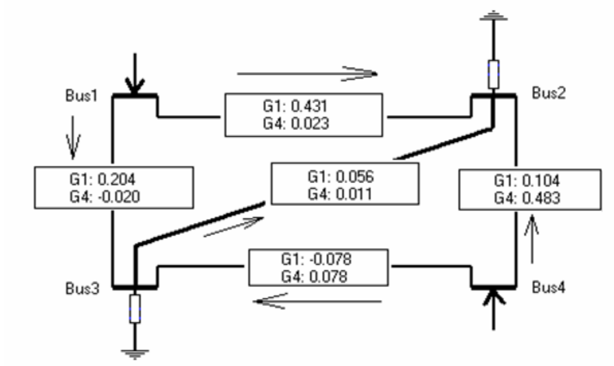

Fig. 6. The loss tracing result of 4-bus test system

The real power flow tracing result obtained from the method proposed in [7] and [13] is shown in Figure 7. Several differences between Figures 5 and 7 can be found. In Figure 7, the result shows that a generator may only inject power to certain line. For example, the real line flows from bus 4 to bus 3 and bus 4 to bus 2 contributed by generator bus 1 are zero. While in Figure 5, the real line flows from bus 4 to bus 3 and bus 4 to bus 2 contributed by generator bus 1 are –0.954 MW and 0.954 MW, respectively.

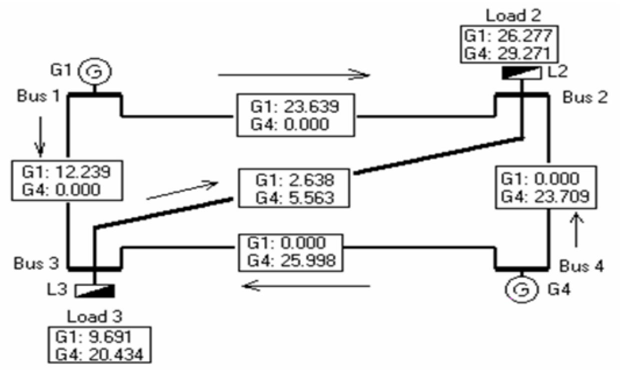

Fig. 7. Power tracing result using Bialek's Method [7], [13]

This paper also shows the result of IEEE-14 bus system. IEEE-14 bus system consists of two generators and three synchronous condensers. Load flow analysis including bus data and line power flows for this test system is given in Appendix. This test system also indicates the injected MVAR that will influence the test result. In [2], there is no injected MVAR is considered. Even though the value of injected MVAR is small for this test system, however it will affects of overall calculation. Table 3 shows the result of real (MW) and reactive (MVAR) power contribution from individual generators to line flows and losses in MW. From this table we can see that there are effects of real power from bus 3, bus 6 and bus 8 even though by looking the power flow solution, the real power is zero for these three buses. Table 4 shows the load tracing results of IEEE-14 bus system. From this table, if we sum up all the power allocation that contributed from all generators, the result is equal to the result that we obtained from power flow solution (see Appendix). This shows the correctness of the methods that presented in this paper.

4. CONCLUSION

In this paper, the superposition theory method for power flow and loss allocation is presented. The method can determine the amount of real and reactive power output from particular generator to a particular load. The loss allocation of each line, which is produced by each generator, can also be obtained. In addition, this paper also take the consideration of injected MVAR in the network system into the calculation of the power contribution. The algorithm is simple and accurate. Accordingly, two test system were selected as test case to show simplicity and veracity of the method. The method couls be used to resolve some difficult pricing to ensure fairness in the power system industry.

Table 3. Result of contribution of individual generators to line flows and its losses of IEEE 14-bus test system

Line ID	Generator Bus 1			Generator Bus 2			Generator Bus 3			Generator Bus 6			Generator Bus 8		
	Real P (MW)	Reactive Q (MVAR)	Line Loss	Real P (MW)	Reactive Q (MVAR)	Line Loss	Real P (MW)	Reactive Q (MVAR)	Line Loss	Real P (MW)	Reactive Q (MVAR)	Line Loss	Real P (MW)	Reactive Q (MVAR)	Line Loss
line 1-2	162.061	-14.460	1.697	-4.639	-7.191	1.255	0.560	-1.118	0.658	-0.673	1.450	0.323	-0.313	0.884	0.372
line 1-5	70.501	-1.301	1.453	4.639	7.191	-0.206	-0.560	1.118	0.301	0.673	-1.450	0.668	0.313	-0.884	0.557
line 2-3	60.971	-9.219	1.824	11.741	13.766	-0.811	2.118	-6.667	1.407	-0.931	3.074	-0.081	-0.583	2.596	-0.009
line 2-4	44.714	-10.055	0.750	10.695	11.414	-0.223	-0.421	1.519	0.225	0.336	-0.902	0.407	0.638	-1.980	0.510
line 2-5	32.601	-6.099	-0.140	8.869	10.218	0.011	-0.658	1.893	0.201	0.663	-2.389	0.462	0.239	-1.424	0.382
line 3-4	-20.206	7.723	-0.881	-3.531	-2.274	0.356	-0.134	8.935	0.744	0.399	-4.394	0.094	0.267	-4.898	0.071
line 4-5	-50.543	19.171	0.139	-8.144	-5.203	0.051	-0.649	1.474	0.167	0.919	-6.021	-0.050	-1.335	2.463	0.176
line 4-7	20.256	-9.034	-0.160	4.942	3.451	-0.344	0.089	2.743	-0.151	0.198	-2.072	0.105	1.666	-11.165	0.549
line 4-9	12.594	-3.474	0.098	2.587	2.438	-0.236	-0.166	1.613	-0.128	-0.071	-1.186	0.097	0.557	-2.178	0.169
line 5-6	32.196	-17.391	0.305	8.207	4.597	-0.859	0.642	3.588	-0.402	3.938	-10.654	0.922	0.773	-0.674	0.034
line 6-11	8.804	2.483	0.172	0.552	2.176	-0.004	-0.551	0.532	-0.016	0.135	5.125	-0.034	-0.726	-1.825	-0.002
line 6-12	7.110	-0.738	0.122	1.130	1.480	-0.004	-0.204	0.726	-0.014	0.190	1.307	-0.016	-0.183	0.349	-0.008
line 6-13	17.095	0.113	0.369	2.281	3.783	-0.008	-0.660	1.619	-0.039	0.107	3.993	-0.058	-0.529	0.264	-0.015
line 7-8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	-21.608	0.000
line 7-9	24.796	-1.529	0.423	3.897	5.516	-0.090	-0.860	2.943	-0.093	-0.845	-1.981	0.041	0.163	10.430	-0.281
line 9-10	3.686	-1.222	0.008	0.893	0.801	-0.002	-0.065	0.711	-0.003	-0.475	-3.761	0.013	0.418	2.950	-0.010
line 9-14	7.107	-2.288	0.133	1.571	1.369	-0.011	-0.064	1.008	-0.022	-0.314	-1.889	0.037	0.395	2.366	-0.046
line 10-11	-5.262	-2.261	0.071	-0.114	-1.356	-0.003	0.394	-0.182	-0.006	-0.224	-4.603	-0.012	0.655	2.064	-0.003
line 12-13	1.819	0.234	0.011	0.182	0.425	0.001	-0.096	0.142	-0.001	0.120	0.713	0.000	-0.160	-0.155	-0.001
line 13-14	6.242	0.740	0.148	0.625	1.417	-0.002	-0.298	0.439	-0.013	0.296	3.283	-0.034	-0.474	-1.050	0.001

Table 4. Load tracing result of IEEE 14-bus system

Load Bus No.	Generator Bus 1		Generator Bus 2		Generator Bus 3		Generator Bus 6		Generator Bus 8	
	Real P (MW)	Reactive Q (MVAR)	Real P (MW)	Reactive Q (MVAR)	Real P (MW)	Reactive Q (MVAR)	Real P (MW)	Reactive Q (MVAR)	Real P (MW)	Reactive Q (MVAR)
2	22.078	1.916	2.799	5.258	-1.136	2.129	-1.064	1.686	-0.979	1.71
3	79.354	-21.083	16.084	14.863	0.845	10.234	-1.248	7.483	-0.84	7.502
4	35.672	-21.287	9.764	5.05	0.882	4.563	0.62	3.904	0.863	3.869
5	6.572	-1.661	1.275	1.249	-0.096	0.74	-0.084	0.643	-0.066	0.629
6	11.188	1.468	1.213	2.716	-0.595	1.096	-0.212	1.228	-0.388	0.996
9	28.427	1.557	3.599	6.61	-1.266	2.912	-0.765	2.644	-0.495	2.877
10	8.939	1.027	1.009	2.153	-0.456	0.893	-0.265	0.847	-0.227	0.882
11	3.3	0.054	0.444	0.749	-0.135	0.34	-0.043	0.34	-0.066	0.317
12	5.17	-1.064	0.953	1.015	-0.095	0.576	-0.087	0.572	-0.014	0.502
13	12.292	-0.658	1.845	2.672	-0.418	1.297	-0.011	1.331	-0.199	1.163
14	13.069	-1.771	2.209	2.693	-0.327	1.429	-0.02	1.336	-0.035	1.312

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APPENDIX

Table A-1. Bus data of IEEE 14-bus test system

Bus No.	Voltage Mag.(V)	Angle (Degree)	Load MW	Load Mvar	Generation MW	Generation Mvar	Injected Mvar
1	1.0600	0.0000	0.0000	0.0000	232.5719	-15.7590	0.0000
2	1.0450	-4.9865	21.7000	12.7000	40.0000	46.1735	0.0000
3	1.0100	-12.7397	94.2000	19.0000	0.0000	25.7260	0.0000
4	1.0147	-10.2597	47.8000	-3.9000	0.0000	0.0000	0.0000
5	1.0176	-8.7690	7.6000	1.6000	0.0000	0.0000	0.0000
6	1.0700	-14.4302	11.2000	7.5000	0.0000	21.9583	0.0000
7	1.0488	-13.2538	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0849	-13.2565	0.0000	0.0000	0.0000	22.3577	0.0000
9	1.0328	-14.8338	29.5000	16.6000	0.0000	0.0000	0.1900
10	1.0318	-15.0474	9.0000	5.8000	0.0000	0.0000	0.0000
11	1.0472	-14.8555	3.5000	1.8000	0.0000	0.0000	0.0000
12	1.0537	-15.2974	6.1000	1.6000	0.0000	0.0000	0.0000
13	1.0469	-15.3194	13.5000	5.8000	0.0000	0.0000	0.0000
14	1.0210	-16.0746	14.9000	5.0000	0.0000	0.0000	0.0000
Total			259.0000	73.5000	272.5719	100.4566	0.1900

Table A-2. Line parameter data of IEEE 14-bus test system

Line No.	From	To	R (pu)	X (pu)	B (total)
1	1	2	0.01938	0.05917	0.05280
2	1	5	0.05403	0.22304	0.04920
3	2	3	0.04699	0.19797	0.04380
4	2	4	0.05811	0.17632	0.03740
5	2	5	0.05695	0.17388	0.03400
6	3	4	0.06701	0.17103	0.03460
7	4	5	0.01335	0.04211	0.01280
8	4	7	0.00000	0.20912	0.00000
9	4	9	0.00000	0.55618	0.00000
10	5	6	0.00000	0.25202	0.00000
11	6	11	0.09498	0.19890	0.00000
12	6	12	0.12291	0.25581	0.00000
13	6	13	0.06615	0.13027	0.00000
14	7	8	0.00000	0.17615	0.00000
15	7	9	0.00000	0.11001	0.00000
16	9	10	0.03181	0.08450	0.00000
17	9	14	0.12711	0.27038	0.00000
18	10	11	0.08205	0.19207	0.00000
19	12	13	0.22092	0.19988	0.00000
20	13	14	0.17093	0.34802	0.00000