Estimation of Monthly Average Hourly and Daily Global Irradiation in Togo

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ABSTRACT

Solar irradiation data recorded in three localities (Lomé, Atakpamé and Mango) representing the climate and geographical areas in Togo are analyzed.

The new technique proposed recently by Jain using normal distribution curve Eq. 1 for estimating the long range averages of instantaneous (or hourly) global radiation has been tested and found to fit hourly global radiation data fairly well in three zones in Togo. The values are obtained by matching the experimental and the theoretical values at the solar noon or at 13 hours according to the months in the year. These σ values are found to be related to \overline{N} by linear equation with excellent coefficients of correlation.

Monthly daily global radiation on a horizontal surface for the three towns, are deduced from their relative sunshine data using the method developed by Angstrom. Appropriate regional parameters are determined and use to predict solar irradiation in all the three localities with an error less than 8%.

1. INTRODUCTION

In order to appraise the economics of a proposed solar-energy application in a particular area and to design a conversion device which will meet a potential demand, it is necessary to know the amount of solar radiation obtainable at the pertinent location. It has involved long term measurements of the instantaneous (or hourly) global irradiation on a horizontal surface in most countries in Africa, however in relatively few meteorological stations. For places where it is not directly measured, hourly and daily solar radiation can be estimated by interpolation from nearby localities where radiation data are available by using models and empirical correlations.

The first attempt to analyze the hourly radiation data was done by Whillier [1] and Hottel and Whillier [2] who used the data of widely separated localities to obtain the curves of the ratio of the hourly/daily global radiation (r_i) in terms of the sunset hour angle for each hour. Collares-Pereira and Rabl [3] developed an analytical expression for the ratio (r_i) in terms of the sunset hourly angle. Recently, Jain [4,5] used the measured monthly average hourly global irradiation data and showed that a normal curve could be fitted to this data fairly closed for all the months. In this paper, the authors are using the Jain's technique to analyze the monthly averages hourly global radiation recorded in some meteorological stations in Togo.

The model most used to predict the daily radiation is the correlation found by Angstrom [6], Black, et al. [7] and others, between global radiation and the duration of sunshine, which is measured at many meteorological stations. The Angstrom's formula locally set up in this work is used to predict the monthly average daily global radiation on a horizontal surface in three localities in Togo.

The present work stems from the need of knowledge of solar radiation data in Togo and to fill the gap of radiation data for the place lacking direct radiation measurements.

2. METHODOLOGY

2.1 Measurement Procedure

Global solar radiation was measured in three localities, whilst several stations recorded sunshine duration in Togo. These are Lome southern coastal belt, Atakpamé in the central region and Mango in the northern savanna region representing the geopraphical and climate zones in Togo, a West-African small country. The geographical locations of the three towns are shown in Table 1.

Solar global radiation was measured by Eppley pyranometer and sunshine duration by a Campbell-Stock tropical sunshine recorder. The data used in this paper are monthly average of hourly and daily global solar radiation on a horizontal surface from 1981 to 1992, and sunshine duration hours averages over many years. These were supplied by Togo meteorological services.

2.2 Analysis of Hourly Global Irradiation Using Jain's Method

Essentially, in this technique the ratio of the hourly/daily irradiation is given by the normal distribution curve [4, 5]:

$$p(t) = [1/\sigma\sqrt{2\pi}] \exp\left[-(t - T_{\star})^2/2\sigma^2\right]$$
(1)

The unknown parameter for any month is determined by equating the value of p(t) for $t = T_o$ with that of the measured value, i.e.:

$$p(t) = [1/\sigma\sqrt{2\pi}]$$
 or $\sigma = 1/p(T_e)\sqrt{2\pi}$ (2)

Stations	Latitude 🛛	Longitude	Altitude
Lomé	06°10' North	01°15' Bast	19,60 m
Atakpamé	07°35' North	01°07' East	499,66 m
Sokodé'	08°59' North	01°09' East	386,48 ∎
Kara*	09°33' North	01°10' East	339,66 🖿
Mango	10°22' North	00°28' East	144,70 m

Table 1. Geographical locations of towns used in the study.

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 T_{\circ} is solar standard time corresponding to the mean of monthly average hourly radiation. In fact, the data show that the mean radiation during the day is reached at solar noon or at 13 hours according to the months in the year as shown in Table 2.

Having determined σ for each of the months and for each locality, Eq. 1 is used to yield the theorical values of the hourly/daily ratio for any hour centered at the solar standard time t.

To know the value of σ for the place where no measured values of the hourly irradiation are available, a linear correlation was developed between σ and \overline{N} :

$$\sigma = a_{\rho} + b_{\rho} \overline{N} \tag{3}$$

The values of the average day length \overline{N} for a given month is computed from:

$$N = 2/15 \cos^{-1} \left(-\tan\phi \tan\delta_c\right) \tag{4}$$

where

$$\delta_c = 23.45 \sin \left[360 \left(284 + D \right) / 365 \right] \tag{5}$$

The local parameters a_{a} and b_{a} are found for each locality.

2.3 Global Solar Radiation and Sunshine Hours Correlation

Of the many models in the literature, the most popular is the regression equation of Angstrom [6].

$$H/H_{a} = a + b\left(\overline{n}/\overline{N}\right) \tag{6}$$

The above equation is used to determine a and b for three localities and also the atmospheric transmission for cloudless skies T = a + b by setting $\overline{n}/\overline{N} = 1$.

The monthly daily global solar radiation from each of the three stations are calculated from Eq. 6 (Table 3). In these calculations the values of \overline{H}_{o} are calculated according to [8]:

$$H_o = 24I_o/\pi \left[\cos\phi\cos\delta_c\sin\omega_s + (2\pi\omega_s/360)\sin\phi\sin\delta_c\right]$$
(7)

where

$$I_{\rho} = 1353 \{1 + 0.033 \cos [360 (D/365)]\}$$
(8)

$$\cos \omega_{e} = -\tan \phi \tan \delta_{c} \tag{9}$$

The least square method is used to calculate the coefficient a and b for different locations.

3. RESULTS AND DISCUSSION

3.1 Hourly Values

The measured values of the 11-year monthly average hourly global irradiaiton for Lomé, Atakpamé and Mango are available at laboratory. The ratio hourly/daily is easily calculated using

Months	t	7	8	9	10	11	12	13	14	15	16	17	18
	Lomé	15.4	96.7	214.0	376.3	500.6	591.9	611.7	553.7	439.4	289.0	136.7	29.7
Jan.	Atakpamé	13.4	107.2	264.8	422.8	551.6	629.4	637.2	578.6	461.8	305.8	146.0	35.0
	Mango	19.4	119.0	284.3	442.5	565.7	644.8	657.0	598.2	483.5	331.8	164.0	36.7
	Lomé	20.7	127.8	286.0	440.4	591.4	714.3	747.4	683.0	547.4	359.8	178.7	44.4
Feb.	Atakpamé	16.6	126.0	299.6	476.4	616.6	696.2	703.6	617.8	488.4	326.4	167.6	46.0
	Mango	19.2	133.5	309.3	489.5	610.5	697.0	718.3	661.8	547.5	386.1	204.8	55.1
	Lomé	35.2	164.0	325.0	496.4	644.4	750.2	761.6	685.6	549.6	372.8	186.0	49.2
March	Atakpamé	27.8	153.6	330.6	511.2	651.8	721.6	708.2	636.2	494.4	317.8	152.6	45.4
	Mango	31.7	155.1	330.3	498	623.1	705.1	713.0	649.8	535.5	375.1	201.1	57.5
	Lomé	55.0	200.2	374.4	562.0	716.0	800.4	792.4	718.0	575.8	390.6	194.6	47.6
April	Atakpamé	60.2	213.6	398.4	555.8	655.2	695.4	699.4	626.2	497.2	315.0	150.4	41.0
	Mango	53.6	184.4	346.2	506.8	622.0	687.2	698.8	624.4	508.8	351.6	189.4	52.0
	Lomé	68.5	210.7	439.8	533.3	632.3	705.0	719.8	670.0	538.6	380.0	199.6	49.8
May	Atakpamé	71.6	229.0	401.8	551.4	650,2	682.4	647.4	579.6	489.8	336.4	177.6	48.8
	Mango	66.5	196.0	352.7	499.3	601.6	649.0	645.7	590.5	489.4	353.1	197.1	60.3
June	Lomé	48.2	157.2	303.0	427.4	503.4	570.8	604.4	568.2	507.6	334.4	176.6	52.4
	Atakpamé	55.6	190.0	356.2	488.0	585.2	629.8	608.0	569.6	444.8	314.4	180.0	54.0
	Mango	67.3	182.1	319.0	440.0	528.3	598.3	617.0	574.8	488.8	367.8	213.5	76.1
July	Lomé	45.5	141.2	285.5	404.5	597.5	691.2	620.7	592.5	516.7	375.2	212.2	63.7
	Atakpamé	34.5	128.1	264.5	408.6	505.3	520.3	529.5	478.1	411.1	296.6	172.1	64.3
	Mango	49.0	149.1	276.1	387.5	464.6	528.5	549.3	514.5	459.3	360.6	209.5	81.1
	Lomé	37.6	141.4	269.0	390.6	498.2	584.8	637.8	537.6	527.6	381.0	202.6	52.0
Aug.	Atakpamé	29.2	111.8	248.6	391.0	485.6	530.6	526.6	475.8	397.8	283.2	148.0	44.4
	Mango	42.3	142.3	263.3	372.0	461.5	514.1	544.8	506.0	437.6	332.5	196.0	73.6
	Lomé	50.5	184.1	345.3	465.6	613.8	706.8	758.6	705.8	587.0	411.6	203.6	39.0
Sep.	Atakpamé	41.5	153.8	312.1	450.1	531.3	576.8	563.6	553.3	450.3	425.6	154.5	32.3
	Mango	44.6	166.5	310.0	438.1	541.0	615.0	627.6	570.0	475.3	337.5	173.6	58.5
	Lomé	79.6	238.0	409.3	550.1	691.3	776.1	776.5	701.5	557.8	351.3	150.5	18.0
Oct.	Atakpamé	71.3	237.1	423.0	562.3	630.6	670.0	662.3	595.8	455.6	302.5	129.3	16.3
	Mango	65.0	214.0	378.1	520.6	623.6	668.6	542.8	591.1	466.5	313.0	132.5	19.3
	Lomé	61.5	256.5	384.6	535.3	651.5	763.3	690.5	658.3	488.6	294.5	122.8	16.1
Nov.	Atakpamé	1										103.1	
	Mango	+		+		+						96.0	1
	Lomé	28.1			+	1						5 105.5	1
Dec.	Atakpamé		+		1							5 90.0	1
Dec.	Mango	26.6	-					3 596.					

Table 2. Long-term average values (in Wh/m² day) of the measured hourly global radiation in three localities.

	Lo	né	Atak	pamé	Mango		
Months	\overline{N}	Ø	\overline{N}	Ø	\overline{N}	۵	
January	11.69	2.54	11.62	2.60	11.69	2.62	
February	11.81	2.60	11.77	2.62	11.81	2.68	
March	11,96	2.63	11.96	2.72	11.96	2.73	
April	12.14	2.71	12.6	2.80	12.14	2.80	
May	12.28	2.74	12.34	2.84	12.28	2.90	
June	12.35	2.78	12.42	2.88	12.35	2.95	
July	12.32	2.75	12.38	2.88	12.32	2.93	
August	12.20	2.72	12.24	2.83	12.20	2.84	
September	12.00	2.67	12.00	2.78	12.00	2.77	
October	11.86	2.61	11.83	2.70	11.86	2.70	
November	11.72	2.60	11.66	2.64	11.72	2.64	
December	11.65	2.55	11.60	2.60	11.69	2.62	
	o = 0.023	3 N + 2.48	σ = 0.03	1 N + 2.5	σ=0.03·	4 N + 2.5	
	r=0	.98	r=0).97	r=0	.99	

Table 3. Long-term average σ and \overline{N} values.

the values from Table 2. For the three localities, these values are plotted against the solar standard time for January, April and August, separately, and are shown by dotted lines in Fig. 1 to Fig. 9.

Following the technique, σ values were calculated for each of the months and for each locality. Finally, using these values of σ and Eq. 1, the theoretical curves were plotted. These are shown in Fig. 1 to Fig. 9 by solid lines.

Since the \overline{N} values of most of localities are recorded and compiled by the meteorological services, the σ values were related to \overline{N} values. The σ and \overline{N} values are presented in Table 4 and the values of σ were plotted against \overline{N} values as shown in Fig. 10 to Fig 12.

Table 4.	Comparison of measured and estimated values of monthly aver	age
	daily global radiation by Eq. 1.	

		Lomé			Atakpamé			Mango		
Months	<i>Н</i> _м	H _s	error (%)	\overline{H}_{M}	Η _ε	error (%)	H _M	\overline{H}_{s}	error (%)	
January	3845.0	4398.4	-14.4	4154.4	4228.6	-2	4382.7	4279.9	2,3	
February	4375.1	4497.6	-2.8	4582.8	4706.5	-2.9	4974.7	4735.9	4.8	
March	4743.1	4957.1	-4.5	4753.2	4550.3	4.0	5052.8	4971.9	1.6	
April	5191.1	4905.5	5.5	4911.8	4723.2	3.6	5275.0	5016.5	4.9	
May	5094.8	4732.6	7.1	4870.6	4684.6	3.5	4791.4	4780.8	0.2	
June	4218.5	4129.2	2.1	4479.6	4278.8	4.2	4488.3	4419.3	-0.7	
July	4142.8	4123.6	0.4	3818.6	4014.7	-5.3	3995.7	4074.8	-1.9	
August	4331.3	4285.8	1.0	3676.0	3762.8	-2.5	3904.9	3981.1	-1.9	
September	4551.1	4296.2	5.6	4065.2	4071.6	-0.3	4327.5	4180.0	3.4	
October	5150.0	4851.3	5.8	4761.6	4410.3	7.1	4630.4	4608.7	0.4	
Nove n ber	4058.5	3900.2	3.9	4588.6	4635.9	-1.2	4271.7	4485.9	5.0	
December	3873,0	4341.2	-12.0	3733.8	4137.2	-11	3931.1	4107.4	-4.4	
Average	4464,5	44510.5	0.29	4366.3	4350.3	0.36	4502.15	4470.18	0.50	



Fig. 1. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for January in Lome.



Fig. 2. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for January in Atakpame.



Fig. 3. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for January in Mango.



Fig. 4. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for April in Lome.





Fig. 5. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for April in Atakpame.



April - Mango

Fig. 6. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for April in Mango.



Fig. 7. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for August in Lome.



Fig. 8. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for August in Atakpame.

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Fig. 9. Comparison of the experimental (dotted line) and the theoretical (solid line) curves for the global radiation for August in Mango.



Fig. 10. Correlation between σ and \overline{N} for global radiation in Lome.

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Fig. 11. Correlation between σ and \overline{N} for global radiation in Atakpame.



Fig. 12. Correlation between σ and \overline{N} for global radiation in Mango.

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Using the least square technique, the linear equations below are obtained to fit the data with an excellent correlation.

$\sigma = 0.023\overline{N} + 2.48$	r = 0.98	for Lome	(10)
$\sigma = 0.031\overline{N} + 2.50$	r = 0.97	for Anakpanie	(11)
$\sigma = 0.034\overline{N} + 2.50$	r = 0.99	for Mango	(12)

These correlations are fairly different for the three localities. In fact the values obtained using the three equations turn out to be quite close to each other. However, to average out the little differences in the three equations, Eq. 13 which is obtained by averaging the coefficients of the three equations, is recommended for estimating the hourly global irradiation at a place in Togo where such measurements are not available.

$$\sigma = 0.029N + 2.49 \tag{13}$$

As may be seen from Fig. 1 to Fig. 9, the agreement between the experimental and the theoretical curves are fairly good for the three localities. These analyses concern all the months in the three localities even if all the figures are not presented.

For nearly four-fifths of the day around the solar noon, the agreement remains good with an average magnitude of error about 4% to 8% for all the months. Only near the sunrise and the sunset hours when the agreement is poor. Jain has shown that this is due to the fact that a normal curve in principle extends up to infinity; so it does not attain the zero values at the sunrise and the sunset hours [5]. However, it may be pointed out that most of the radiation during the day is confined within about two-thirds or three-forths of the day around the solar noon [9]. Therefore, the discrepancy around the sunset and sunrise hours is of little practical significance.

3.2 Daily Values

The variation of monthly solar radiation for the three towns are shown in Fig. 13. They are similar in pattern. Peak insolation occurs in March-April and then in October-November. These values are less important than those estimated by Thompson [10] using the data of only ten stations for all the African continent.

For Mango, the northern savanna town, the annual average of daily irradiation is about 4.5 kWh/m^2 per day, contrary to 4.3 kWh/m^2 per day at Atakpamé and 4.4 kWh/m^2 per day in Lomé.

The predicted average values \overline{H}_{E} was calculated using Eq. 6, the regression coefficients *a* and *b* and their correlation coefficients for each station are presented in Table 3 and Table 5. It is seen that the correlation is good for Mango and Atakpamé except for Lomé, where r = 0.73. It shows a better accuracy for all the months with an exception of Lomé where the discrepancy reaches 14% in December and January. In general, the predicted values show an excellent agreement with the measured values in the three towns and the remainder of the months throughout the year, the difference being less than 8%.

Table 5 shows that the sum a + b which represents the clear day fraction of \overline{H}_{o} increases both southwards and northwards from the central region as well as the sunshine fraction $\overline{n}/\overline{N}$.

Stations	$\overline{n}/\overline{N}$ annual average	а	b	a + b	r
Lomé	0.550	0.263	0.374	0.637	0.73
Atakpané	0.532	0.253	0,357	0.610	0.86
Sokodé*	0.58	0.297	0.188	0.485	0.88
Kara*	0.579	0.257	0.300	0.557	0.72
Mango	0.583	0.271	0.300	0.571	0.93

Table 5. Regression and correlation coefficients.

* obtained from the reference [12]



Fig. 13. Variation of monthly average daily global radiation \overline{H} .

Figure 14 shows the variation of monthly daily clearness index $\overline{H}/\overline{H}$, for the three towns throughout the year. Peak clearness index occurs in March-April and then in October-November. The minima are observed in June-July-August corresponding to the middle of the rainy season in the three zones.



Fig. 14. Variation of monthly average daily clearness index K_r

4. CONCLUSION

The normal distribution curve Eq. 1 recently suggested by Jain is found to fit the data well with an average magnitude of error about 4% to 8% for all the months. The agreement is poor near the sunrise and the sunset hours and an approach to make the technique almost perfect or accurate has to be outlined.

The σ values computed are related to \overline{N} values by linear correlation. Table 4 and Fig. 10 to Fig. 12 show that the correlation is good in each locality.

On the other hand, it is possible to compute global solar radiation on a horizontal surface in Togo using Eq. 13, Eq. 14, and Eq. 15, respectively for Lome, Atakpame, and Mango and for the nearby localities where radiation data are not available.

$$\bar{H}/\bar{H}_{a} = 0.263 + 0.374 \,(\bar{n}/\bar{N})$$
 (13)

$$\bar{H}/\bar{H}_{o} = 0.253 + 0.357 \,(\bar{n}/\bar{N})$$
 (14)

$$\bar{H}/\bar{H}_{a} = 0.271 + 0.30 \,(\bar{n}/\bar{N})$$
 (15)

It is therefore possible to determine clear-day global solar irradiation at any location in the country using the appropriate coefficients a + b in Table 5.

Table 3 indicates that apart from some few months in the year, the error in the computed values using Eq. 13 to Eq. 15 is expected to be in the range of $\pm 8\%$.

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6. NOMENCLATURE

a,, b,	=	climatologically determined regression coefficients for hourly radiation
a, b	=	
		climatologically determined regression coefficients for daily radiation.
D	=	the average day of the year given for each month
Ħ	=	monthly daily radiation received on a horizontal surface (kWh/m ² per day)
Ħ,	=	monthly daily radiation on a horizontal surface in the absence of any atmosphere
-		at a particular latitude (kWh/m ² per day)
\overline{H}_{R}	=	predicted monthly daily radiation received on a horizontal surface
2		(kWh/m ² per day)
I,	=	solar corrected constant (W/m ²)
κ _τ		$\overline{H}/\overline{H}_{o}$, the clearness index
p(t)	=	the theoretical ratio of the hourly/daily radiation
r	=	correlation coefficient
r_{i}	=	the ratio hourly/daily for the observed global radiation
t	=	solar standard time in hours
T n	=	solar standard time corresponding to the mean of daily radiation (hours)
n	=	monthly average daily number of hours of observed bright sunshine (hours/day)
N	=	monthly average daily maximum possible sunshine duration (hours/day)
σ	=	standard deviation parameter in the normal distribution
Sc	=	characteristic delination, the declination on which the extra-terrestrial irradiation
		is identical to its monthly average value [11]
φ	=	latitude in degrees
	=	sunset hour angle in degrees
ω,	_	subset nour angle in degrees

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