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# Real-Coded Genetic Algorithm Applied to Optimal Placement of Capacitor Banks for Unbalanced Distribution Systems with Meshed/Radial Configurations 

R. Hooshmand* and M. Ataei ${ }^{* 1}$


#### Abstract

One of the most important methods in loss reduction and controlling the voltages of distribution systems is the utilization of the fixed and switched capacitors. To do this, real modeling of the system in large scale unbalanced or balanced for both radial and meshed configurations are required. In this paper, a new technique for finding the optimal values of the fixed and switched capacitors in the distribution networks based on the Real-Coded Genetic Algorithm (RCGA) is presented. In this method, the modeling of radial or loop feeders with unbalanced or balanced network loads are basically considered. Also, the modeling of the load at different levels is simulated which low voltage and medium voltage capacitors those are available in the market are used. Regarding the above factors in addition to the various parameters in optimization problem, the RCGA is used to find the best and real optimal network with the best rate for the capacitors. Finally, this methodology is tested on a region of the distribution network of the city of Ahvaz in Iran and satisfactory results are obtained. These results show that in addition to the decreasing of the network losses and improvement of the voltage profile, the benefit saving due to application of capacitors is increased.


Keywords - Unbalanced Distribution Networks, Optimization, Genetic Algorithm, Capacitor Placement.

## 1. INTRODUCTION

One of the most effective and useful methods in reducing the power losses of distribution networks is utilization of optimal capacitors. By using shunt capacitors, the reactive power needed for loads are provided, so that besides the reduction of losses, the voltage profile of nodes is also improved. There are, of course, numerous difficulties in optimal placement of capacitors in the purpose of reducing losses. These problems include: a) non-clarity of the behavior of feeders' loads, particularly domestic loads, b) complexity of distribution networks, c) uncertainty of electric distribution companies in returning the initial capital expenditure used for capacitor placement and d) variety of the type of network loads. With due regard to these difficulties, in many researches made so far, some assumptions in capacitor placements have been considered to solve the problem in a more simple method [1]-[20], which have not been appealing to distribution companies, so that the losses are still high in the network.

Most of the consumption loads in distribution networks are single phase domestic loads which are unbalanced. Therefore, it is useful to investigate the capacitor placement for unbalanced distribution networks [1]-[3]. However, it has not been regarded in the most of the previous works [4]-[20]. On the other hand, in the methods presented in [6]-[15], and [17], the loss reduction has been

[^0]accomplished only by using the fixed capacitors. Moreover, in many previous methods, the medium voltage capacitors which are more expensive than low voltage ones are used [2], [3], and [5]-[19].

One of the important issues in placement of capacitors is considering the load variations of the network. In some methods [2], [3], [5], [15]-[17], and [19]-[20], load variation has been considered at several different levels, and in some other methods [1], [6]-[14], and [18], it has not been considered at all, and the load has been presented in a fixed form. Moreover, Capacitor placement is also advised to be done with the daily real value in the market, so that the distribution companies may be assured of its productivity. This has been taken into consideration but only in references [3], [6], and [16].

In the previous works presented by researchers, capacitor placement has been done on the basis of different techniques including: integer programming method [2], mixed linear integer programming method [20], nonlinear programming method [1], [3], method of sensitivity analysis [12], [16], and [19], method of optimization of the Equal Area Criterion for selecting the sites of fixed capacitors [13], dynamic programming method [8], and some methods based on the experimental criteria. In these methods in order to solve the capacitor placement problem, some assumptions have been considered on the type of the objective function and also on the type and number of problem restrictions. There is also the difficulty of trapping the answer of the problem in a local optimal solution. Moreover, since the capacitor banks contain discontinuous values, solving the problem in continuous domain and then approximating the results leads to a large error in optimal solution. With due regard to the above problems, the genetic algorithm is a very useful tool in solving the optimization problems [4]-[11], [17], and [18].

In this research, the objective is to find the optimal location and values of the fixed and switched capacitors in distribution networks by using Real Coded Genetic Algorithm. The important characteristics of this new method are:
a) Network modeling for both balanced and unbalanced load cases.
b) Considering the meshed and/or radial configurations for distribution networks which are less considered in previous papers.
c) Utilization of fixed and switched capacitors available in the market and at low and medium voltages with their real prices.
d) Utilization of the load model of the network at different load levels.
e) Using of standard values of capacitors in proposed method based on the RCGA instead of using the estimated values.
For this purpose, at first, the modeling of the network elements is explained in second section. Then, the direct load flow analysis in unbalanced or balanced cases for meshed and/or radial configurations is briefly presented in section 3. In section 4, after a brief review of real-coded GA, the algorithm for designing fixed and switched capacitors will be presented. Finally, in order to test the presented algorithm, this algorithm has been implemented on feeder No. 3062 in Kian-Pars region of Ahvaz, which well acceptable results show the effectiveness of the proposed algorithm.

## 2. MODELING OF THE NETWORK ELEMENTS

In the optimal capacitors placement, in order to reduce the losses and to control the network voltage, all the practical aspects and the realities dominating the distribution network should be taken into account. These issues include the precise modeling of network and loads, performing the unbalanced three-phase load flow with radial and/or meshed configuration, and utilizing real usable capacitors, which will be discussed in the coming sections.

## Network Modeling

In order to design a practical algorithm, a proper model of the network is required. For this purpose, the great care should be taken to model the network properly, and the inductance and resistance of the three-phase lines with due regard to the variety of the wires used in the distribution network are considered in the program. Moreover, it should be possible to consider intricate sub-branches and meshed network in the algorithm under discussion. All these points have been taken into consideration in the proposed method.

## Three-phase Load Modeling in the Distribution Network

In distribution networks, wide ranges of various electric loads such as domestic loads, industrial trade loads, and street lamps are encountered. Besides, each of these loads has lots of fluctuations within a year which create many
difficulties in modeling of the load in the networks. One of the important methods to overcome this problem, which need little information in this respect, is to extend the use of load fluctuations of the feeding post to the consumers' loads. For this purpose, at first, the curve of the three-phase feeder's 24 hour active load should be extracted from the post, then the load variation diagram should be drawn in the form of percentage of load peak versus the percentage of consumption time. Fig. 1 shows an example of this diagram, which indicates that the feeder's load is $50 \%$ of the peak load in $62.5 \%$ of 24 hours, and it forms $80 \%$ of the peak load in $2.1 \%$ of 24 hours. Now this three-phase load distribution of the post can be extended to all the three-phase loads of feeder's branches.

Thus, in this method the variations of load with a good approximation by using little information have been considered.


Fig. 1. Load variation in the form of percentage of the peak load in terms of percentage of time.

## Real Modeling of Capacitors

In finding the optimal values of capacitors, the problem should be designed in such a way that utilization of the range of capacitors available in the market of the electric industries may become possible. Otherwise, rounding of the amount of the designed capacitors to the nearest value of the available capacitor will not minimize the nonlinear cost function. To overcome this problem in this algorithm, at first, a list of available capacitors in the market is prepared, then, a suitable combination with due regards the purchasing costs, installation, and maintenance is formed and used as an alphabet collection for selecting the genes of every chromosome in real coded genetic algorithm to design fixed and switchable capacitors. This problem is discussed in the next sections.

In finding the values of capacitors, the selection criterion should not be the capacitors' fixed reactive powers because their reactive power varies by variation of the feeder's voltage. Therefore, the capacitor modeling should be performed on the basis of fixed impedance. In other words,
the reactive power of every capacitor $\left(Q_{C}\right)$ is calculated in each stage of load distribution repeatedly by using the following equation:

$$
\begin{equation*}
Q_{c}=\frac{V_{c}^{2}}{X_{c}} \tag{1}
\end{equation*}
$$

Another important problem in placement of capacitors which should be considered is the rate of growth of investment costs for designing of capacitors. Because, from the economical point of view an acceptable design is one whose productivity is more than the initial investment costs plus the growth rate of initial costs. Therefore, the initial costs including the costs of purchase, erection, lateral equipment and maintenance must be multiplied by the coefficient for capital turnover. That is,

$$
\begin{equation*}
A_{c}=C_{c} \cdot\left[\frac{i(1+i)^{\mathrm{n}}}{(1+i)^{\mathrm{n}}-1}\right] \tag{2}
\end{equation*}
$$

In this equation, $i$ stands for annual rate of growth of money, and $n$ for the life-span of the project (in years). It should be noted that annual average prices due to maintenance can be added to value of $A_{C}$. Finally, it can be said that by real modeling of available capacitors in the market and by considering the growth rate of money, all the relevant economic problems have been taken into consideration, so that the system directors may be quite satisfied about purchasing capacitors

## 3. THREE-PHASE LOAD FLOW ANALYSIS

In this section a direct power flow method with meshed/ radial configurations for balanced and unbalanced distribution networks is explained [23].

## Solution for Networks with Radial Lines

In this method, the special topology of distribution networks has been fully exploited to make direct solution possible. Also, by applying Kirchhoff's laws and inserting the source bus in all equations by representing the loads as constant impedances, a set of voltage equations can be written as the following matrix form:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{s}}=\mathrm{Z} . \mathrm{I} \tag{3}
\end{equation*}
$$

In other words, the number of above mentioned equations is equal to the number of system loads. Therefore, this equation is in a matrix form that each row represents the KVL equation which is written between bus source and the load corresponding to that load. In the above equation, $\mathbf{V}_{8}$ is the column vector which each element is voltage of the source node and $V_{s}=1 \mathrm{pu}$. Moreover, $\mathbf{I}$ is a column vector which each element is the current of one load node of the network. $\mathbf{Z}$ is the impedance matrix of the network [23]. By solving equation (3), the current vector of the nodes load of the network is determined and then all nodes voltages can be calculated.

## Solution for Networks with Radial and Loop Lines

Distributions networks that have both radial and loop lines are at first converted into a radial system and then by the presented method for radial networks are solved. Fig. 2a. shows a typical distribution networks that have both redial and meshed lines and Fig. 2b. shows the equivalent redial system after breaking the meshed nodes.


Fig. 2. Distribution Network with both Meshed and Radial Lines; a) before and b) after Mesh Breaking.

The procedure of breaking the loop lines and making an equivalent radial system is shown in Fig. 3. As shown in this figure, $i_{p 1}$ and $i_{p 2}$ are the currents through path 1 and path 2 which the sum of them flows in the load impedance $Z_{\text {load }}$. Therefore, the following equations can be written:

$$
\begin{align*}
& V_{s}=Z_{\text {com }} \cdot\left(i_{p 1}+i_{\rho 2}\right)+Z_{\rho 3 \text { bit } 1} \cdot i_{p 1}+Z_{\text {boad }} \cdot\left(i_{p 1}+i_{p 2}\right)  \tag{4}\\
& V_{s}=Z_{\text {com }} \cdot\left(i_{p 1}+i_{p 2}\right)+Z_{p a n 2} \cdot i_{p 2}+Z_{\text {ioad }} \cdot\left(i_{p 1}+i_{p 2}\right)
\end{align*}
$$

where $Z_{\text {com }}, Z_{\text {path1 }}, Z_{\text {path } 2}$ and $Z_{\text {load }}$ are the impedances of common path of the current, path1, path2 and load, respectively. Equation (4) can be rearranged as follows:

$$
\begin{align*}
& V_{s}=\left(Z_{\text {com }}+Z_{\text {pacin } 1}+Z_{\text {ioad }}\right) \cdot i_{p 1}+\left(Z_{\text {com }}+Z_{\text {boad }}\right) \cdot i_{\rho 2} \\
& V_{\mathrm{s}}=\left(Z_{\text {com }}+Z_{\text {boad }}\right) \cdot i_{p 1}+\left(Z_{\text {com }}+Z_{\text {path } 2}+Z_{\text {ioad }}\right) \cdot i_{\rho 2} \tag{5}
\end{align*}
$$

Thus, the loop node $i$ is braked into the nodes $i_{1}$ and $i_{2}$. In fact, the meshed network is converted into two radial
networks. Finally, after load flow analysis and determining the voltage and current values of these nodes, as shown in Fig. 3, the voltages of these nodes is equal to the voltage of the $i$ node and it can be written:

$$
\begin{align*}
& V_{i 1}=V_{i 2}=V_{i} \\
& V_{i}=Z_{\text {load }} \cdot\left(i_{p 1}+i_{p 2}\right) \tag{6}
\end{align*}
$$



Fig. 3. Calculation of Loop Node's Current.

## Solution for Unbalanced Networks

In unbalanced distribution networks, the model of equivalent impedance of the lines and network components are represented as single phase form for each node in $3 \times 3$ matrices. The information of the loads powers are presented and modeled for each phase separately. As a result, the presented method for balanced networks can be used also for unbalanced three-phase networks which lead to well acceptable results.

Because of three-phase representation, each load node will represent three loops. For the $n$th load node, $[3(n-1)+1]$ th, $[3(n-1)+2]$ th and $[3(n-1)+3]$ th loops will represent the $a, b$ and $c$ phases, respectively. Formation of the $\mathbf{Z}$ matrix is as simple as in the single-phase case. Because of the three-phase representation, network identity between the $i$ th and $(i+1)$ th load nodes will appear in the $[3(i-1)+1]$ and $[3(i+1-1)+1]$ th rows of the $\boldsymbol{Z}$ matrix. Similar to the balanced case, there is no need for inverting the impedance matrix and the load distribution problem can be solved by using the LU factorization.

## 4. TECHNIQUE OF DESIGN BASED ON THE GENETIC ALGORITHM

## The Structure of Real-Coded Genetic Algorithm (RCGA)

The Genetic Algorithm initiates the mechanism of the natural selection and evolution and aims to solve an optimization problem with object function $f(x)$ where $x=\left\lfloor x_{1}, x_{2}, \cdots, x_{N}\right\rfloor$ is the N -dimensional vector of optimization parameters. It has proved to be an effective and powerful global optimization algorithm for many combinatorial optimization problems, especially for those problems with discrete optimization parameters, nondifferentiable and/or discontinuous objective function [22].

Genes and chromosomes are the basic building blocks of the GA. The conventional standard GA (SGA) encodes the optimization parameters into binary code string. A gene in SGA is a binary code. A chromosome is a concatenation of genes that takes the form

$$
\begin{align*}
\text { chromosome }= & {\left[g^{1} g^{1} \mathbf{g}_{2}^{1} \ldots g_{L_{1}}^{1}\right.}  \tag{7}\\
& \left.\ldots g_{1}^{N} g_{1}^{N} g_{2}^{2} \ldots \ldots g_{2}^{2} \ldots g_{L_{L N}}^{2}\right]=\left[x_{1} \ldots X_{2} \ldots \ldots . x_{N}\right]
\end{align*}
$$

where $g_{j}^{i}$ is a gene, and $L_{i}$ is the length of the code string of the $i$ th optimization parameter and

$$
x_{k}=\left[\begin{array}{llll}
g_{1}^{k} & g_{2}^{k} \cdots & g_{L k}^{k} \tag{8}
\end{array}\right]
$$

The genetic algorithm used in this paper is RCGA. Real number encoding has been confirmed to have better performance than either binary or gray encoding for constrained optimization problems [21]. Then, in the RCGA, a gene is the optimization parameter itself where is selected from Alphabet set. The chromosome takes the form:

$$
\text { chromosome }=\left[\begin{array}{lll}
x_{1} & x_{2} & \ldots x_{N} \tag{9}
\end{array}\right]
$$

The RCGA structure is summarized as follows:

## Initial population

The RCGA operates on a population of $N_{p o p}$ chromosomes simultaneously. The initial population of real numbered vectors is created randomly. Each of these vectors represents one possible solution to the search problem. The population size ( $N_{p o p}$ ) generally varies from 2 to 2.5 times the number of genes.

Once the initialization is completed, the population enters the main GA loop and performs a global optimization for searching the optimal solution of the problem. In a GA loop, the stages 2 to 7 are carried out in turn. The GA loop continues until the termination conditions in stage 3 are fulfilled.

## Scaling

The scaling operator, a preprocessor, is usually used to scale the object function into an appropriate Fitness Function. It aims to prevent premature convergence in the early stages of the evolution process and to speed up the convergence in the more advanced stages of the process.

## Termination criterion

After the fitness has been calculated, it has to be determined if the termination criterion has been met. This can be done in several ways. The algorithm used here stops when a finite generation number has been reached and the best fit among the population is declared the winner and solution to the problem.

## Selection

The selection (or reproduction) operator selects good chromosomes on the basis of their fitness values and produces a temporary population, namely, the mating pool. This can be achieved by many different schemes, but the
most common method is Roulette Wheel Selection. The roulette wheel is biased with the fitness of each of solution candidates. The wheel is spun $M$-times where $M$ is the number of strings in the population. This operation generates a measure that reflects the fitness of the previous generation's candidates.

## Crossover

The crossover operator is the main search tool. It mates chromosomes in the mating pool by pairs and generates candidate offspring by crossing over the mated pairs with probability $P_{\text {cross }}$. The probability of parent-chromosome crossover is assumed to be between 0.6 and 1.0. Many variations of crossover have been developed, e.g. one-point, two-point and $N$-point, and random multipoint crossover. Here, the arithmetical one-point crossover is used and introduced.

## Mutation

After crossover, some of the genes in the candidate offspring are inverted with the probability $P_{\text {mut }}$. This is the mutation operation for the GA. The mutation operator is included to prevent premature convergence by ensuring the population diversity. A new population is therefore generated. In this paper, the probability of mutation ( $P_{m u t}$ ) is assumed to be between 0.01 and 0.1 .

## Elitism

The postprocessor is the elisit model. The worst chromosome in the newly generated population is replaced by the best chromosome in the old population if the best number in the newly generated population is worse than that in the old population. It is adopted to ensure the algorithm's convergence. This method of preserving the elite parent is called elitism.

## The Design Flowchart of the Fixed Capacitor

The design flowchart of fixed capacitors by utilizing RCGA has been shown in Fig. 4. In this method, the initial three-phase load flow for both balanced and unbalanced cases is done without any capacitors such that the annual energy loss of three-phases may be calculated ( $2^{\text {nd }}$ stage of flowchart). In this stage the constraints of load flows and, specially upper and lower limits of each node magnitude voltage are investigated. If a node voltage is not within limits, it is more probable to insert capacitor in that node. Then the primary population is selected at random (stage 3 ). The number of genes in each chromosome of the population is equal to the number of network nodes. The value of each gene which is the value of capacitor in that node is selected randomly from the alphabet set. The alphabet set is a vector which its components are the values of KVA of capacitors available in market. Then for every chromosome, the capacitors exist in every gene are injected into the system and the three-phase load flow is performed (stage 4) such that the Annual Energy Loss (AEL) of the system can be obtained for each chromosome as follows:
$A E L=\sum_{\mathrm{i}=1}^{\text {NLL }}\left[\binom{\right.$ power loss in }{ load level $i} *\binom{$ time percentage of }{ year for load level $\left.i} * 365 * 24\right]$
and the Average Daily Energy Loss (ADEL) is calculated as follows:

$$
\begin{equation*}
A D E L=\frac{A E L}{365} \tag{11}
\end{equation*}
$$

In order to calculate the value of saving resulted from capacitor placement for every chromosome, the difference of annual loss of energy with and without using capacitors is calculated. Then, the amount of reduced energy loss is multiplied by the cost for every kWh . The difference between the annual cost of the used capacitors and the saving resulted from capacitor placement, is the annual net benefit (ANB) of capacitor placement for each chromosome, which is considered as the objective function. In other words,

$$
\begin{align*}
& A N B=\left[\binom{A E L \text { with }}{\text { capacitors }}-\binom{A E L \text { without }}{\text { capacitors }}\right] \\
& *(\text { cost of every kWh })-\binom{\text { The annual cost }}{\text { of capacitors }} \tag{12}
\end{align*}
$$

It should be noted that $A N B$ function can be considered as the objective function of problem. Thought it seems that this function represents the net benefit of capacitor placement, however, minimization of energy loss and capacitors prices are considered implicit. Then, by normalizing the amount of the objective function for each chromosome, the fitness value for each chromosome is obtained (stage 5). Furthermore, for each chromosome by three-phase load flow analysis, the voltage of all the nodes is evaluated so that the voltage of nodes may not be less than its authorized limit. If the voltage of all the nodes is not within their authorized limits, the related chromosome does not appear in the new population.

Now, if the related population is not converged, on the basis of the selection mechanism that presented in section 4.1, the new population is created. The crossover and mutation operators are performed on this population (stages $7,8,9$ ) so that finally the new population is prepared to repeat the process (stage 10).

By converging of the population on the basis of the best amount of fitness for every chromosome in the population, it can be concluded that optimal capacitor placement in the purpose of the loss reduction and network voltage control with real available capacitors in the market has been satisfactory performed.

## Designing of Fixed and Switched Capacitors

In the distribution networks where load variations in different seasons of year are intensive and network is in the unbalanced status, utilization of switched capacitors in a peak load beside the fixed capacitors have economic justifications. Of course, in the networks where load variations of the feeders during the year are not very intensive, if the fixed capacitors are designed properly, switched capacitors are not economically justifiable. This is
due to that the control and switching of this type of capacitors, in addition to the difficulties created in exploitation, will increase exploitation and capacitor placement investment costs in a peak load. For the threephase networks with high load variations in each phase, the flowchart for finding the optimal location and value of switched capacitors in a peak load beside the fixed capacitors designed by utilization of RCGA have been shown in Fig. 5.

The basic differences of this flowchart with the flowchart of fixed capacitor placement is that the switched capacitors have only been considered for peak load and naturally the three-phase load flow analysis is done only at the peak load level (stage 4). Moreover, during the design of switched capacitors, the optimal amount of fixed ones, which has been designed based on the algorithm presented in Fig. 4, is permanently and constantly considered in the network. It should be noted that in stage (5) in determining of the fitness value of each chromosome, the annual costs of capacitors include costs of both fixed capacitors and switched ones.


Fig. 4. Flowchart for Solving the Case of Fixed Capacitor Placement by using Real-Coded Genetic Algorithm.


Fig. 5. Flowchart of Solving the Problem of Switched and Fixed Capacitors by Utilizing RCGA.

## 5. CASE STUDY

## Network Specifications and Initial Data

In this simulation, it has been considered that the rate of investment growth to be $10 \%$, the cost of energy per kWh is 100 Rials (Iranian currency) and life span of capacitors placement for 30 years is assumed. Also, the $N_{p o p}$ is considered as 2.5 times the number of genes, $P_{\text {cross }}$ equals to 0.8 and $P_{m u t}$ is assumed to be between 0.01 and 0.1 . This simulation has been performed on feeder No. 3062 in Kian-Pars region in Ahvaz/Iran. The percentage of load variation of the consumers of this feeder versus time percentage has been shown in Table 1. The single line diagram of this feeder has been shown in Fig. 6 which by connecting the node No. 5 to No. 18 and the node No. 45 to No. 67, the network is converted to meshed/radial configuration. In this figure, the sign $\otimes$ stands for location of distribution transformers $11 \mathrm{kV} / 400 \mathrm{~V}$ and $\bullet$ indicates a node on whose basis no capacitor placement is possible. With due regard to the fact that capacitor placement is done by capacitors available in the market; the cost of low voltage capacitors available in the local market has been shown in Table A of the Appendix A.


Fig. 6. The Single Line Diagram of the Feeder No. 3062 in Kian-Pars of Ahvaz.

Table 1. Load Percentage of Feeder No. 3062 in Proportion to its Time Percentage

| Percentage of maximum load | Time percentage of year |
| :---: | :---: |
| $100 \% \mathrm{P}_{\max }$ | $42 \%$ (equal to 5 months) |
| $55 \% \mathrm{P}_{\max }$ | $29 \%$ (equal to 3.5 months) |
| $25 \% \mathrm{P}_{\max }$ | $29 \%$ (equal to 3.5 months) |

The specifications and parameters of the lines of this feeder and the maximum loading $\left(\mathrm{P}_{\max }\right)$ of distribution transformers have been shown in Table $B$ of the Appendix B.

## The Capacitor Placement Design for Unbalanced Network

In this network, in nodes 21, 30, and 59 single-phase load is considered on phase $a$, in nodes 22,31 , and 60 singlephase load is put on Phase $b$, and in nodes 23, 32, and 61 single-phase load is considered in phase $c$. Therefore, in spite of having 9 nodes with single-phase load on the different phases, the network is in an unbalanced status and it is desired to minimize the network loss. In this section, the simulation results are provided for three different cases including: without capacitor placement, with fixed capacitors, and mixed fixed and switched capacitors placement.

## Case I: The network status without capacitor placement

Since the network loads during a year have been considered in three different load levels, in the first case, three-phase load flow in these levels without any capacitor placement is accomplished. The minimum voltage in three phases $a, b$, and $c$ in three different load levels is provided in Table 2 which is related to the node No. 69. It should be noted that the first load level is the peak loaded and is $P_{\text {max }}$, the second load level is intermediate (or normal) loaded and is $0.55 P_{\text {max }}$, and the third load level is light loaded which is $0.25 P_{\max }$. By calculating the loss of these three load flows, the average daily energy loss for phases $a, b$, and $c$ are
$938.47 \mathrm{kWh}, 914.08 \mathrm{kWh}$, and 928.88 kWh respectively, which the mean value of total daily energy loss becomes 2775.43 kWh . The total amount of the daily energy consumption in three phases is equal 138765.3 kWh and the power factor of the feeder in the load peak is 0.825 . These results are presented in Table 3. In addition to the node No. 69 which has the least value of voltage, 14 nodes of 72 feeder nodes have also the voltage less than 10.50 kV that shows the undesired voltage profile for this feeder.

## Case II. With fixed capacitor placement

In the implementation of the algorithm presented in section 4.2, it does not have economic justification to use medium voltage fixed capacitors at level 11 kV . This is due to the large number of transformers and the short length of lines, which cause the annual costs of capacitor placement to exceed cost of loss reductions per year in the network, and this makes the project uneconomical. With due regard to the low price of the low voltage capacitors, it is more economical to use them in the secondary positions of distribution transformers. By implementing the fixed capacitors design algorithm, the resulted values of capacitors are as shown in Fig. 7 and the simulation results are provided in the second column of Table 3. By using these designed values of capacitors, the three-phase $A D E L$ per year has decreased from 2775.43 kWh to 2135.13 kWh . The average value of daily energy consumption decreases from 138765.3 kWh to 129207.16 kWh . On the other hand, the daily cost of used capacitors with the values shown in Fig. 7, is 19723.16 Rials. By comparison, it is found that an average daily saving of 25336.32 Rials is achieved. The importance of this type of optimal capacitor placement will be more appeared when we find that by using this method, the profile of the system's voltage has improved favorably. To show this, the network's three-phase load flow is done with optimal fixed capacitor placement. It is found that the minimum voltage in the peak load level and for three phases in bus No. 69 has increased from 10.4744 kV to 10.5727 kV which is an indication of improvement of the voltage profile in the entire network.

Table 2. The Minimum Voltage of Unbalanced Three-Phase Network without any Capacitors

|  | First Load Level <br> (Peak Loaded) | Second Load Level <br> (Normal Loaded) | Third Load Level <br> (Light Loaded) |
| :--- | :---: | :---: | :---: |
| Minimum voltage in phase $a$ | 10.4735 | 10.8105 | 10.8920 |
| Minimum voltage in phase $b$ | 10.4765 | 10.8193 | 10.8986 |
| Minimum voltage in phase $c$ | 10.4741 | 10.8130 | 10.8941 |
| The average of minimum <br> voltage in three-phases | 10.4744 | 10.8143 | 10.8949 |

Table 3. The Simulation Results in Different Cases of Capacitors Placement for Unbalanced Network

|  | Case I: <br> Without <br> Capacitors | Case II: <br> With Fixed <br> Capacitors | With Fixed and <br> Switched <br> Capacitors |
| :--- | :---: | :---: | :---: |
| Minimum voltage in peak load of phase $a$ | 10.4735 | 10.5698 | 10.6887 |
| Minimum voltage in peak load of phase $b$ | 10.4765 | 10.5763 | 10.6913 |
| Minimum voltage in peak load of phase $c$ | 10.4741 | 10.5701 | 10.6901 |
| Average of minimum voltage in peak load of three-phase | 10.4744 | 10.5727 | 10.6900 |
| Average daily energy loss in phase $a$ (kWh/day) | 938.47 | 732.55 | 1346.45 |
| Average daily energy loss in phase $b$ (kWh/day) | 914.08 | 698.96 | 1323.49 |
| Average daily energy loss in phase $c(\mathrm{kWh} /$ day) | 928.88 | 703.62 | 1336.76 |
| Average daily energy loss in three-phases (kWh/day) | 2775.43 | 2135.13 | 4006.7 |
| Average daily energy consumption in three-phases (kWh/day) | 138756.3 | 129207.16 | 215876.09 |
| Daily cost of used capacitors (Rials/day) | ----- | 19723.16 | 22428.56 |
| Average daily saving (Rials/day) | ----- | 25336.32 | 95813.03 |

## Case III. With fixed and switched capacitor placement

Regarding that in this feeder the maximum load of the network during summer, late spring and early autumn has been very high in Ahvaz, and in winter months this reaches $30 \%$ of the maximum load, so it seems that utilization of switched capacitors in parallel with fixed capacitors in this type of network is economical. Therefore, the design algorithm of fixed and switched capacitors in this feeder was performed. The designed values of switched capacitors in the peak time of the load have been shown in Fig. 8 and third column of the Table 3 as well. By placing switched capacitors beside fixed capacitors in the peak load level of the feeder, the average daily energy loss in the peak load has reached to 4006.6 kWh and the daily energy consumption also is reduced to 215876.09 kWh . It should be noted that the above mentioned values are related to the peak load level; therefore the daily energy loss in this level is more than the average daily energy loss per year. The feeder's power factor in the peak load is also increased to 0.973 , which indicates the improvement in the status of the distribution network in the case of using fixed and switched capacitors simultaneously. Furthermore, with due regard to the price of the fixed and switched capacitors and the amount of the reduced loss of energy in different load levels, the average daily saving is 95813.03 Rials in the peak load. The total saving in a year will be 21287342.8 Rials which is a considerable quantity for a feeder in a region of Ahvaz.

Furthermore, regarding the voltage profile of the nodes in the peak load and in the presence of fixed and switched capacitors it is found that the minimum voltage of the network in node No. 69 in this state has increased to 10.6900 kV . Thus, the network voltage control in addition to the loss reduction
and well acceptable economical justifications, have been achieved in a favorable manner.

## The Capacitor Placement Design for Balanced Network

In order to complete the examination of this case study, the network in the balanced status with radial configuration is considered. This configuration is achieved by switching off the connection nodes 5 to 18 and also nodes 45 to 67 . The simulation results are provided in Table 4 which shows the minimum network voltage without capacitor placement in the peak load is equal 10.4687 kV that is less than allowable limit. Fixed capacitor placement with the values presented in Fig. 9, causes the reduction of the average daily energy loss from 2990.38 kWh to 2476.27 kWh . Regarding the daily cost of capacitor placement, which equals 22796.87 Rials/ day, it is found that total average of daily saving is 28614.06 Rials/day.

Since the load variation of the network in the peak, normal and light load levels is high, it is concluded that switched capacitors besides the fixed ones in the peak load may be economically useful. Therefore, the switched capacitors are designed as Fig. 10. In this case, the average daily energy loss reaches 4353.66 kWh in the peak load. Regarding the cost of the daily capacitor placement for both fixed and switched capacitors equal 25575.27 Rials/day, the average daily saving in the peak load of the network is 108425.6 Rials/day and total saving per year is equal 22096624.10 Rials, which is considerable for each year. Moreover, minimum voltage in the peak load is increased to 10.6566 kV which improves the voltage profile of the network.


Fig. 7. The Results of Fixed Capacitors Placement for Unbalanced Network in Feeder No. 3062.


Fig. 8. The Results of Switched Capacitors Placement for Unbalanced Network in Feeder No. 3062.


Fig. 9. The Results of Fixed Capacitors Placement for Balanced Network in Feeder No. 3062.


Fig. 10. The Results of Switched Capacitors Placement for Balanced Network in Feeder No. 3062.

Table 4. The Simulation Results in Different Cases of Capacitors Placement for Balanced Network

|  | Case I: <br> Without Capacitors | Case II: <br> With Fixed <br> Capacitors | Case III: <br> With Fixed and Switched <br> Capacitors |
| :--- | :---: | :---: | :---: |
| Average of minimum voltage in peak load of three-phase | 10.4687 | 10.5565 | 10.6566 |
| Average Daily energy loss in three-phase (kWh/day) | 2990.38 | 2476.27 | 4353.66 |
| Average Daily energy consumption in three-phases (kWh/day) | 150358.89 | 149844.66 | 234616.13 |
| Daily cost of used capacitors (Rials/day) | ------ | 22796.87 | 25575.27 |
| Average daily saving (Rials/day) | ------ | 28614.06 | 108425.65 |

## 6. CONCLUSION

In this paper, a new method for optimal capacitor placement in distribution networks in the unbalanced case with meshed/ radial configuration based on Real-Coded Genetic Algorithm was presented. With due regard to the many constraints and the practical restrictions in distribution networks, application of the RCGA in order to find a real optimal and applicable solution for the quantity and place of fixed and switched capacitors is very effective. At first, it is tried to model the network and also to consider the balanced and unbalanced feeder's loads at different levels so that the load modeling may be performed with minimum assumptions. Moreover, by considering the real model and prices of capacitors which are available in the market, it was found that the fixed and switched capacitor placement can be executed at the low voltage level ( 380 V ) and medium voltage level ( $11 \mathrm{kV}, 20 \mathrm{kV}$ and 33 kV ). Then, all above mentioned facts were considered in the presented design algorithm of fixed and switched capacitors. At the end, this algorithm was implemented on the feeder No. 3062 in the city of Ahvaz in Iran.

The obtained results indicate that the presented procedure of capacitor placement based on RCGA has a realistic view to this important practical problem and causes the reduction in the energy loss and annual energy consumption in the network which is attractive in the economic point of view. It has also improved the feeder's nodes voltage and controls the voltage in a favorable manner.

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## APPENDIX A. Specifications of Low Voltage Capacitors

The range of low voltage capacitors available in the market and their prices is wide and various, but in this simulation, the norm quantities of capacitors available in the market with their approximate prices have been utilized according to Table A.

Table A. Specifications of Low Voltage Capacitors

| Capacitor <br> Number | Capacity <br> (kVAR) | Nominal <br> Voltage <br> (V) | Approx. price including <br> (ateral equipments (Rials) |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 380 | 0.0 |
| 1 | 10 | 380 | 680,000 |
| 2 | 15 | 380 | 850,000 |
| 3 | 20 | 380 | $1,000,000$ |
| 4 | 25 | 380 | $1,200,000$ |
| 5 | 30 | 380 | $1,300,000$ |
| 6 | 40 | 380 | $1,600,000$ |
| 7 | 50 | 380 | $2,000,000$ |

APPENDIX B. The Specification of Feeder No. 3062 in Ahvaz

Feeder No. 3062 in Kian-Pars region of Ahvaz is contains 72 line pieces, which naturally include 72 nodes (without the
node concerning the feeder post). The specifications of these lines with the scale of loading from the distribution transformers in the last node of these lines have shown in Table B.

In this table, the line number, sending and receivering node numbers, resistance ( R ), and reactance ( X ) of the line pieces, the three-phase active and reactive powers received in the peak load by the loads available at the end nodes of the lines have been specified in the term of kW and kVAR.

Table B. The Specifications of Feeder's Lines and Loads in Feeder No. 3062

| Line No. | Send Node | End Node | Impedance $(\mathbf{R}+\mathbf{j X})$ | Load Power in End Node (kW + j kVAR) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | $0.05136+\mathrm{j} 0.09146$ | $97.75+\mathrm{j} 60.58$ |
| 2 | 1 | 2 | $0.03455+\mathrm{j} 0.06153$ | $120.00+\mathrm{j} 63.22$ |
| 3 | 2 | 3 | $0.00467+\mathrm{j} 0.00831$ | $0.0+\mathrm{j} 0.0$ |
| 4 | 3 | 4 | $0.01401+\mathrm{j} 0.02494$ | $160.65+\mathrm{j} 99.56$ |
| 5 | 4 |  | $0.01404+\mathrm{j} 0.02494$ | 160.65 + j 99.56 |
| 6 | 3 |  | $0.04903+\mathrm{j} 0.08730$ | $0.0+\mathrm{j} 0.0$ |
| 7 | 6 | 7 | $0.03362+\mathrm{j} 0.05987$ | $160.65+\mathrm{j} 99.56$ |
| 8 | 7 | 8 | $0.030535+\mathrm{j} 0.05405$ | $0.0+\mathrm{j} 0.0$ |
| 9 | 8 | 9 | $0.00934+\mathrm{j} 0.01663$ | $115.13+\mathrm{j} 71.35$ |
| 10 | 8 | 10 | $0.01588+\mathrm{j} 0.02827$ | $136.04+\mathrm{j} 84.31$ |
| 11 | 10 | 11 | $0.00934+\mathrm{j} 0.01663$ | $192.78+\mathrm{j} 119.47$ |
| 12 | 6 | 12 | $0.02335+\mathrm{j} 0.04157$ | $34.0+\mathrm{j} 21.07$ |
| 13 | 12 | 13 | $0.01167+\mathrm{j} 0.02079$ | $0.0+\mathrm{j} 0.0$ |
| 14 | 13 | 14 | $0.02802+\mathrm{j} 0.04989$ | $217.6+\mathrm{j} 134.86$ |
| 15 | 14 | 15 | $0.00794+\mathrm{j} 0.01414$ | $0.0+\mathrm{j} 0.0$ |
| 16 | 15 | 16 | $0.01261+\mathrm{j} 0.02245$ | 47.6 + j 29.5 |
| 17 | 16 | 17 | $0.00934+\mathrm{j} 0.01663$ | 160.65 + j 99.56 |
| 18 | 17 | 18 | $0.00467+\mathrm{j} 0.00831$ | 160.65 + j 99.56 |
| 19 | 15 | 19 | $0.03269+\mathrm{j} 0.05820$ | $86.7+$ j 53.73 |
| 20 | 19 | 20 | $0.03502+\mathrm{j} 0.06236$ | $127.5+\mathrm{j} 79.02$ |
| 21 | 13 | 21 | $0.01634+\mathrm{j} 0.02910$ | $243.65+\mathrm{j} 151.0$ |
| 22 | 21 | 22 | $0.04529+\mathrm{j} 0.08065$ | $227.58+\mathrm{j} 141.05$ |
| 23 | 22 | 23 | $0.03362+\mathrm{j} 0.05987$ | $110.5+\mathrm{j} 68.48$ |
| 24 | 21 | 24 | $0.01634+\mathrm{j} 0.02910$ | $0.0+\mathrm{j} 0.0$ |
| 25 | 24 | 25 | $0.01494+\mathrm{j} 0.02661$ | $102.0+\mathrm{j} 63.21$ |
| 26 | 25 | 26 | $0.00654+\mathrm{j} 0.01164$ | $144.58+\mathrm{j} 89.61$ |
| 27 | 26 | 27 | $0.04950+\mathrm{j} 0.08814$ | $255.0+\mathrm{j} 158.04$ |
| 28 | 27 | 28 | $0.02802+\mathrm{j} 0.04989$ | 160.65 + j 99.56 |
| 29 | 24 | 29 | $0.01541+\mathrm{j} 0.02744$ | $0.0+\mathrm{j} 0.0$ |
| 30 | 29 | 30 | $0.03502+\mathrm{j} 0.06236$ | $210.8+\mathrm{j} 130.64$ |
| 31 | 30 | 31 | $0.01167+\mathrm{j} 0.02079$ | 155.29 + j 96.24 |
| 32 | 31 | 32 | $0.01401+\mathrm{j} 0.02494$ | 160.65 + j 99.56 |
| 33 | 29 | 33 | $0.01868+\mathrm{j} 0.03326$ | $0.0+\mathrm{j} 0.0$ |
| 34 | 33 | 34 | $0.02802+\mathrm{j} 0.04989$ | $179.39+\mathrm{j} 111.18$ |
| 35 | 34 | 35 | $0.02568+\mathrm{j} 0.04573$ | $232.97+\mathrm{j} 144.37$ |
| 36 | 33 | 36 | $0.01634+\mathrm{j} 0.02910$ | $0.0+\mathrm{j} 0.0$ |
| 37 | 36 | 37 | $0.01201+\mathrm{j} 0.03742$ | 165.65 + j99.56 |
| 38 | 37 | 38 | $0.02802+\mathrm{j} 0.04989$ | 289.0 + j 179.11 |
| 39 | 36 | 39 | $0.00934+\mathrm{j} 0.01663$ | 160.65 + j 99.56 |
| 40 | 39 | 40 | $0.00747+\mathrm{j} 0.01330$ | $0.0+\mathrm{j} 0.0$ |
| 41 | 40 | 41 | $0.03175+\mathrm{j} 0.05654$ | $248.2+\mathrm{j} 153.82$ |
| 42 | 41 | 42 | $0.01868+\mathrm{j} 0.03326$ | 160.65 + j 99.56 |
| 43 | 42 | 43 | $0.01961+\mathrm{j} 0.03490$ | $248.52+\mathrm{j} 154.10$ |
| 44 | 40 | 44 | $0.01634+\mathrm{j} 0.02910$ | 160.65 + j 99.56 |
| 45 | 44 | 45 | $0.03175+\mathrm{j} 0.05654$ | $294.52+\mathrm{j} 182.53$ |
| 46 | 44 | 46 | $0.01681+\mathrm{j} 0.02993$ | $204.0+\mathrm{j} 126.43$ |
| 47 | 46 | 47 | $0.01868+\mathrm{j} 0.03326$ | $321.32+\mathrm{j} 199.13$ |
| 48 | 47 | 48 | $0.03736+\mathrm{j} 0.06652$ | $163.32+\mathrm{j} 101.23$ |
| 49 | 46 | 49 | $0.01634+\mathrm{j} 0.02910$ | $0.0+\mathrm{j} 0.0$ |
| 50 | 49 | 50 | $0.00794+\mathrm{j} 0.01414$ | $204.0+\mathrm{j} 126.43$ |
| 51 | 50 | 51 | $0.02802+\mathrm{j} 0.04989$ | $272.0+\mathrm{j} 168.57$ |
| 52 | 49 | 52 | $0.01027+\mathrm{j} 0.01829$ | $0.0+\mathrm{j} 0.0$ |
| 53 | 52 | 53 | $0.02942+\mathrm{j} 0.05238$ | $238.0+\mathrm{j} 147.5$ |
| 54 | 52 | 54 | $0.01261+\mathrm{j} 0.02245$ | $207.4+\mathrm{j} 128.53$ |
| 55 | 54 | 55 | $0.03129+\mathrm{j} 0.05571$ | $0.0+\mathrm{j} 0.0$ |
| 56 | 55 | 56 | $0.02008+\mathrm{j} 0.03575$ | $176.8+\mathrm{j} 109.57$ |
| 57 | 55 | 57 | $0.02335+\mathrm{j} 0.04157$ | 160.65 + j 99.56 |
| 58 | 52 | 58 | $0.02568+\mathrm{j} 0.04573$ | 160.65 + j 99.56 |
| 59 | 58 | 59 | $0.01728+\mathrm{j} 0.03076$ | $22.1+\mathrm{j} 13.69$ |
| 60 | 59 | 60 | $0.02008+\mathrm{j} 0.03575$ | $144.58+\mathrm{j} 89.61$ |
| 61 | 60 | 61 | $0.00887+\mathrm{j} 0.01580$ | $178.5+\mathrm{j} 110.63$ |
| 62 | 61 | 62 | $0.00560+\mathrm{j} 0.00998$ | 187.0 + j 115.89 |
| 63 | 62 | 63 | $0.00560+\mathrm{j} 0.00998$ | 160.65 + j 99.56 |
| 64 | 63 | 64 | $0.06304+\mathrm{j} 0.11225$ | 238.0 + j 147.50 |
| 65 | 64 | 65 | $0.02335+\mathrm{j} 0.04157$ | $0.0+\mathrm{j} 0.0$ |
| 66 | 65 | 66 | $0.02335+\mathrm{j} 0.04157$ | $30.60+\mathrm{j} 18.96$ |
| 67 | 66 | 67 | $0.01401+\mathrm{j} 0.02494$ | $129.71+\mathrm{j} 80.39$ |
| 68 | 65 | 68 | $0.00467+\mathrm{j} 0.00831$ | $0.0+\mathrm{j} 0.0$ |
| 69 | 58 | 69 | $0.05603+\mathrm{j} 0.09978$ | 160.65 + j 99.56 |
| 70 | 68 | 70 | $0.01401+\mathrm{j} 0.02494$ | 160.65 + j 99.56 |
| 71 | 70 | 71 | $0.00934+\mathrm{j} 0.01663$ | $187.0+\mathrm{j} 115.89$ |
| 72 | 71 | 72 | $0.03829+\mathrm{j} 0.06818$ | $20.4+\mathrm{j} 12.64$ |


[^0]:    *Department of electrical engineering, The University of Isfahan, P.O. Box 81746-73441, Hezar jerib Street, Isfahan, Iran.

    Corresponding author:
    Tel.:+98-311-7933040; Fax: +98-311-7933071;
    E-mail: Ataei@eng.ui.ac.ir

