Methodology of Computing Average Daily Solar Insolation on Inclined Surfaces

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ABSTRACT

Measurements of solar insolation on tilted surfaces, required by several design procedures of solar energy utilization systems are generally not available and it is necessary to rely on computational methods for their estimates. In this paper, methods of calculating the average insolation on inclined surfaces facing directly toward the equator using daily and hourly values are compared. Computation results based on the Hong Kong solar radiation data are presented. Very small differences have been noted between the results obtained by these two methods.

INTRODUCTION

Knowledge on the amount of solar radiation incident on tilted surfaces at different angles is required for the design of flat-plate solar collectors, building cooling load calculation and other applications. Monthly averages of the daily solar radiation incident upon horizontal surfaces are available. However, radiation data on tilted surfaces are extremely rare and one has to rely on various computational methods for their estimates.

A simple method of estimating the monthly average daily radiation on surfaces tilted toward the equator has been developed by Liu and Jordan [1]. An alternative approach to the computation of insolation on tilted surfaces through the use of hourly radiation data has also been investigated by others [2, 3]. A comparison between the two methods using hourly and daily values of solar radiation data to compute the insolation on inclined surfaces has been carried out by Iqbal [4]. In this study, the mathematical formulations of the above two methods are presented and their computation results based on the Hong Kong daily and hourly data are then compared. Only monthly average values of the insolation on tilted surfaces facing directly toward the equator are considered.

METHOD USING DAILY RADIATION VALUES

To compute daily insolation on an inclined surface, one first of all has to separate the diffuse component from the total horizontal solar radiation. If the sky diffuse radiation and the ground reflected diffuse radiation are both assumed to be isotropic, the monthly average daily insolation on an inclined plane, $\overline{H}_\beta$ can be computed from the following expression [1]:

$$\overline{H}_\beta = (\overline{H} - \overline{H}_d)\overline{R}_b + \overline{H}_d\,(1 + \cos\beta)/2 + \rho\overline{H}\,(1-\cos\beta)/2, \quad \ldots\ldots\ldots (1)$$

where $\overline{H}$ = monthly average daily total radiation received on a horizontal surface,

$\overline{H}_d$ = monthly average daily diffuse radiation received on a horizontal surface,

β = angle of tilt of the inclined surface from the horizontal,

ρ = ground reflectance which usually ranges between 0.2 and 0.7,

$\overline{R}_b$ = the ratio of the monthly average beam radiation on the tilted surface to that on a horizontal surface.

The ratio $\bar{R}_b$ is a function of the transmittance of the atmosphere which is in general difficult to evaluate due to the everchanging meteorological and weather conditions. However, Liu and Jordan [1] suggested that $\bar{R}_b$ can be approximated by the ratio of the daily extraterrestrial radiation on a tilted surface to that on a horizontal surface,

$$\bar{R}_b \simeq \bar{H}_o\beta/\bar{H}_o. \qquad \ldots\ldots\ldots (2)$$

For surfaces facing directly toward the equator, $\bar{R}_b$ in equation (2) can be expressed as

$$\bar{R}_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_s' + (\pi/180)\omega_s'\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + (\pi/180)\omega_s\sin\phi\sin\delta} \qquad \ldots\ldots\ldots (3)$$

where ω_s = arcos $(-\tan\phi\tan\delta)$,

ω_s' = minimum of $[\omega_s, \text{arcos}(-\tan(\phi-\beta)\tan\delta)]$,

and ϕ, δ are the latitude and declination of the sun respectively.

When calculating the monthly averages of $\bar{R}_b$ in equation (3), average declination δ values for the month are used. In this study, the 15th day of the month is chosen to calculate the monthly average values.

Actual long term average data measurements for diffuse radiation $\bar{H}_d$ are to be used in equation (1) for computation as far as possible. However, in places where actual measurement data are not available, values of diffuse radiation, $\bar{H}_d$ have to be estimated. Liu and Jordan had provided a correlation between the monthly average diffuse radiation fraction $\bar{H}_d/\bar{H}$ and $\bar{K}_T$ which is the ratio of the average daily total on the earth surface to the extraterrestrial total radiation at the same location. $\bar{K}_T$ is also known as the cloudiness index. A graphical presentation of the above correlation has been given in figure 14 of their paper [5]. Klein [6] later developed the following mathematical expression for Liu and Jordan's correlation:

$$\bar{H}_d/\bar{H} = 1.390 - 4.027\,\bar{K}_T + 5.531\,\bar{K}_T^2 - 3.108\,\bar{K}_T^3 \qquad \ldots\ldots\ldots (4)$$

The monthly average values of the daily diffuse solar radiation in Hong Kong have been calculated by the above correlations with cloudiness index based on the Hong Kong meteorological data [7] for the ten years' period between 1969 and 1978 and the results are presented in Table 1. All radiation data are collected at a station, King's Park located at 22° 19′N, 114° 10′E. The Hong Kong monthly average values of $\bar{H}$, $\bar{H}_o$ and $\bar{K}_T$ for the same period are also shown in Table 1. In Hong Kong the estimated fraction of diffuse radiation may vary between 0.354 and 0.521. Over the year, the highest mean diffuse radiation is 7.39 $MJm^{-2}d^{-1}$ in the summer month of June and the lowest is 4.44 $MJm^{-2}d^{-1}$ in the winter month of December.

Using $\rho = 0.2$ and the estimated values of diffuse solar radiation from Table 1, and values of $\bar{R}_b$ from equation (3), the insolation on inclined planes in Hong Kong is computed by equation (1). Results of the monthly total insolation on inclined surfaces for slopes from 10° to 90° at increments of 10° are shown in Table 2. Figure 1 presents the same results but for four slopes; β = 30°, 50°, 70° and 90°. The insolation on the horizontal surface β = 0° is also shown in figure 1 for comparison. It can be seen that for the summer months between April and September, the insolation on inclined surfaces decreases rapidly with the increasing angles of tilt. However, in the winter months, there seems to be no obvious trends of the insolation variation with the angles of tilt.

Table 1. Calculation of the Hong Kong monthly average diffuse radiation on a horizontal surface by the Liu and Jordan's correlation.

	$\bar{H}_o$ [MJm^{-2}d^{-1}]	$\bar{H}$ [MJm^{-2}d^{-1}]	$\bar{K}_T = (\bar{H}/\bar{H}_o)$	$\bar{H}_d/\bar{H}$	$\bar{H}_d$ [MJm^{-2}d^{-1}]
JAN	25.21	11.48	0.455	0.410	4.71
FEB	29.41	11.76	0.400	0.465	5.47
MAR	33.81	11.95	0.353	0.521	6.23
APR	37.36	13.87	0.371	0.499	6.92
MAY	39.06	16.13	0.413	0.451	7.27
JUN	39.58	17.06	0.431	0.433	7.39
JUL	39.30	18.84	0.479	0.389	7.33
AUG	38.07	18.16	0.477	0.390	7.08
SEP	35.30	17.02	0.482	0.386	6.57
OCT	31.23	15.33	0.491	0.378	5.79
NOV	26.64	13.87	0.521	0.354	4.91
DEC	24.12	12.51	0.519	0.355	4.44

Table 2. Variation of the Hong Kong monthly average insolation on inclined surfaces at various tilt angles [MJm^{-2}d^{-1}]

MONTH	Angles of Tilt (degrees)								
	10	20	30	40	50	60	70	80	90
JAN	12.83	13.90	14.65	15.06	15.13	14.84	14.20	13.24	11.98
FEB	12.59	13.16	13.45	13.45	13.16	12.59	11.75	10.68	9.40
MAR	12.32	12.45	12.33	11.96	11.37	10.56	9.55	8.39	7.11
APR	13.88	13.63	13.11	12.35	11.37	10.19	8.87	7.45	5.98
MAY	15.77	15.11	14.19	13.02	11.64	10.12	8.50	6.88	5.46
JUN	16.46	15.57	14.42	13.04	11.48	9.81	8.10	6.49	5.40
JUL	18.22	17.26	16.00	14.47	12.72	10.82	8.87	7.00	5.56
AUG	17.95	17.37	16.45	15.22	13.71	11.97	10.08	8.12	6.23
SEP	17.39	17.39	17.01	16.27	15.18	13.79	12.13	10.26	8.24
OCT	16.31	16.93	17.16	17.01	16.48	15.59	14.35	12.81	11.02
NOV	15.44	16.65	17.48	17.89	17.88	17.44	16.59	15.34	13.75
DEC	14.27	15.70	16.76	17.42	17.65	17.46	16.83	15.81	14.40

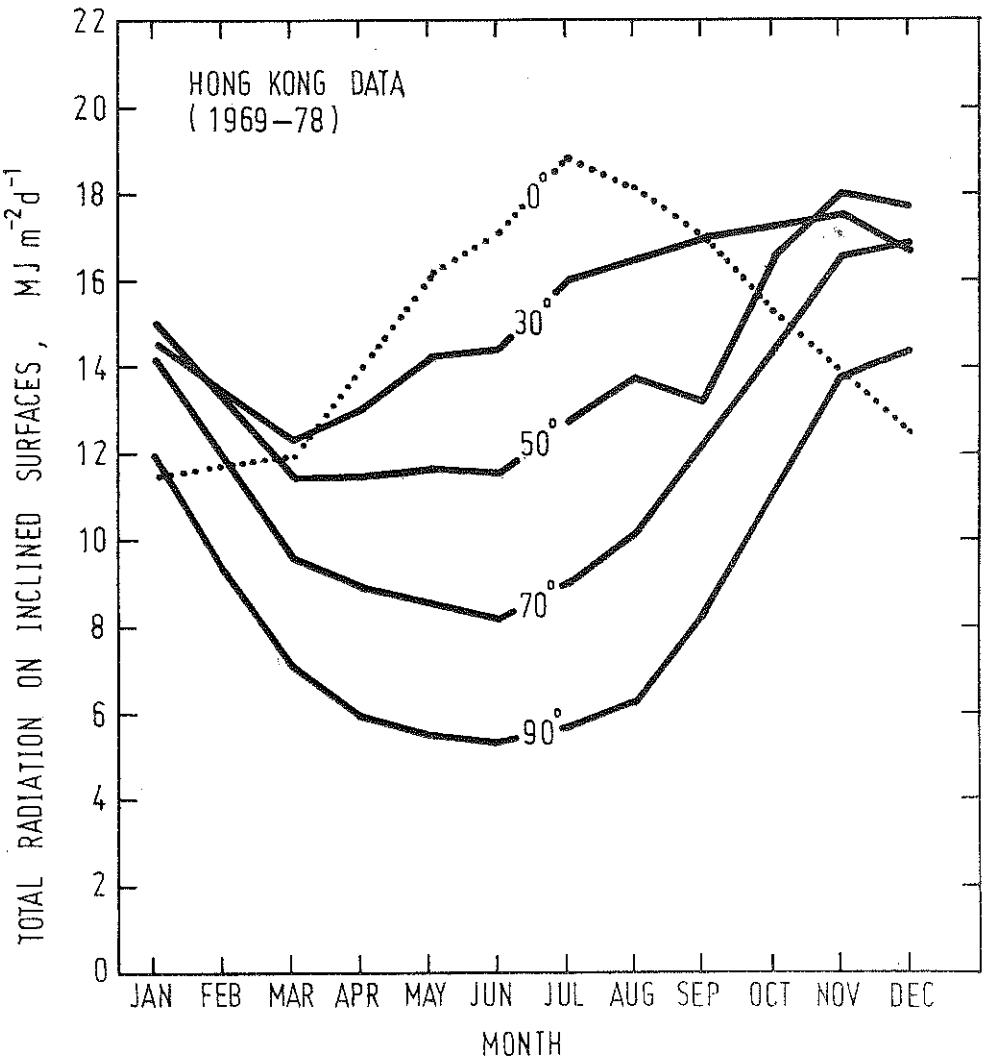

Fig. 1 Monthly variation of total radiation on an inclined plane for different tilt angles.

METHOD USING HOURLY RADIATION VALUES

An alternative method to calculate the average daily radiation on a tilted surface can be obtained by summing the average hourly radiation values on the same tilted surface $\bar{I}_\beta$ from sunrise to sunset.

$$\bar{H}_\beta = \sum_{day} \bar{I}_\beta \qquad \ldots\ldots\ldots (5)$$

The average hourly radiation on a tilted surface $\bar{I}_\beta$ is also composed of three components and if the sky diffuse radiation and the ground reflected diffuse radiation are both assumed to be isotropic, $\bar{I}_\beta$ can be evaluated from the following expression,

$$\bar{I}_\beta = (\bar{I} - \bar{I}_d)\, r_b + \bar{I}_d(1 + \cos\beta)/2 + \bar{I}_\rho\,(1 - \cos\beta)/2 \qquad \ldots\ldots\ldots (6)$$

where $\bar{I}$ = monthly average hourly total radiation received on a horizontal surface,

$\bar{I}_d$ = monthly average hourly diffuse radiation received on a horizontal surface,

r_b = ratio of the hourly extraterrestrial insolation on a tilted surface to that on a horizontal surface,

and ρ, β are the same as defined previously in equation (1).

Considering no change of atmospheric transmission over an hour and for surfaces facing directly toward the equator,

$$r_b = \frac{\cos(\phi-\beta)\cos\delta\cos\omega + \sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta}\ ; \qquad \ldots\ldots\ldots (7)$$

r_b can be evaluated at the mid-point of the hour angle ω and using the same monthly average mean declination value as in equation (3).

To calculate the average hourly insolation on inclined surfaces by equation (6), the data measurements of the mean hourly total and diffuse radiation are required. However, in locations where such hourly data are not available, both the average hourly total, $\bar{I}$ and the diffuse radiation, $\bar{I}_d$ must be estimated from their corresponding daily values.

$$\bar{I} = r_T\,\bar{H} \qquad \ldots\ldots\ldots (8)$$

$$\bar{I}_d = r_d\,\bar{H}_d \qquad \ldots\ldots\ldots (9)$$

The interrelationship between the hourly and daily values of the total and diffuse radiation had been investigated quite thoroughly by Liu and Jordan and values of r_T and r_d were given graphically in their paper [5].

RESULTS AND DISCUSSION

Comparison of the hourly and daily methods of computing insolation on inclined surfaces has been performed by Iqbal [4] using data for three Canadian cities and the correspondance of the results has been very good. In Hong Kong, the recordings of the total global solar radiation on an hourly basis have been obtained only since December 1978 and the results for the twelve months period, December 1978—November 1979 are shown in Table 3. Calculation of the insolation on inclined surfaces using the Hong Kong hourly values for the same twelve months' period, has been carried out and their results are compared with that of using daily values for the same period. Figure 2 shows a typical comparison plot of the Hong Kong results by the above two methods for a few representative slopes of 30°, 60° and 90°. Apart from the slight differences observed, the agreement of the results obtained by the two approaches is generally quite good as seen by the fact that all the data points lie mainly on the 45° slope line.

The existence of some asymmetries in the Hong Kong hourly total radiation data around solar noon appears to be the main reason for the differences between these two results. The daily method implicitly assumes symmetries of diffuse and total radiation around solar noon as represented by the equal sunrise and sunset hour angles of the day. However the hourly method takes into account of the asymmetries of actual data through equation (5). Moreover, the

Table 3. Hong Kong monthly average hourly total radiation received on a horizontal surface ($MJm^{-2}hr^{-1}$)

MONTH	Hours													
	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19
Dec 1978	—	0.06	0.36	0.92	1.52	1.93	2.14	2.16	1.95	1.53	1.00	0.44	0.07	—
Jan 1979	—	0.07	0.33	0.75	1.20	1.55	1.68	1.57	1.40	1.14	0.72	0.30	0.06	—
Feb ,,	—	0.08	0.33	0.70	1.04	1.30	1.47	1.46	1.35	1.10	0.72	0.32	0.07	—
Mar ,,	0.01	0.07	0.21	0.40	0.60	0.79	0.89	0.96	0.92	0.76	0.53	0.27	0.07	0.00
Apr ,,	0.03	0.16	0.43	0.77	1.14	1.44	1.56	1.63	1.47	1.13	0.79	0.48	0.18	0.02
May ,,	0.05	0.22	0.51	0.84	1.13	1.14	1.64	1.66	1.60	1.38	1.05	0.59	0.26	0.05
Jun ,,	0.08	0.33	0.74	1.16	1.57	2.01	2.18	2.08	1.85	1.59	1.23	0.80	0.38	0.09
Jul ,,	0.10	0.41	0.90	1.50	2.14	2.62	2.82	2.80	2.80	2.17	1.65	1.14	0.56	0.12
Aug ,,	0.05	0.29	0.71	1.14	1.57	1.82	1.81	1.75	1.71	1.48	1.11	0.70	0.28	0.04
Sep ,,	0.07	0.37	0.88	1.44	1.89	2.20	2.24	2.10	1.73	1.42	1.02	0.43	0.07	0.00
Oct ,,	0.01	0.16	0.69	1.43	2.11	2.69	3.02	2.96	2.62	2.09	1.34	0.62	0.09	0.00
Nov ,,	—	0.04	0.29	0.74	1.26	1.61	1.81	1.81	1.58	1.25	0.76	0.28	0.02	—

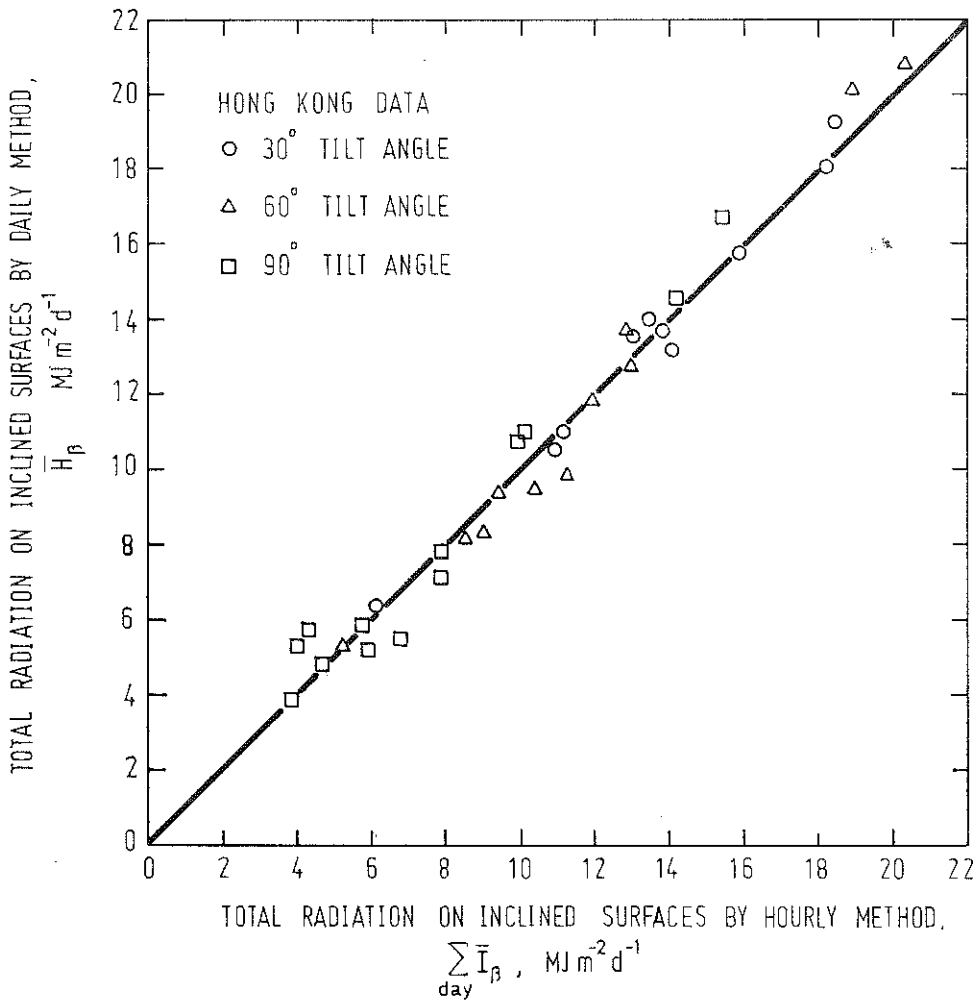

Fig. 2 Comparison of total radiation results on inclined surfaces obtained by the daily and hourly methods.

observed differences between the results do not seem to be restricted to particular angles of tilt. As seen in figure 2, the spread of the data points is almost equal on both sides of the 45° slope line even for different angles of tilt. This indicates that none of the two methods gives consistently higher or lower values of estimates for the insolation on inclined surfaces in Hong Kong.

CONCLUSION

In this paper, computation of the monthly average daily insolation on inclined surfaces facing toward the equator by the daily and hourly methods has been presented and a comparison of results based on the Hong Kong data has been made. The agreement of the results obtained by the two different approaches is in general quite good. The slight discrepancies whenever they arise in some occasions are mainly due to the asymmetries of the hourly radiation data around the solar noon time.

NOMENCLATURE

$\bar{H}$ = monthly average daily total radiation received on a horizontal surface, $MJm^{-2}d^{-1}$

$\bar{H}_d$ = monthly average daily diffuse radiation received on a horizontal surface, $MJm^{-2}d^{-1}$

$\bar{H}_\beta$ = monthly average daily total radiation received on a surface inclined at β degrees to the horizontal, $MJm^{-2}d^{-2}$

$\bar{H}_o$ = extraterrestrial monthly average daily insolation on a horizontal surface, $MJm^{-2}d^{-1}$

$\bar{H}_{o\beta}$ = extraterrestrial monthly average daily insolation on a surface inclined at β degrees to the horizontal, $MJm^{-2}d^{-1}$

$\bar{K}_T$ = ratio of the monthly average total to the monthly average extraterrestrial radiation on a horizontal surface, dimensionless

$\bar{I}$ = monthly average hourly total radiation received on a horizontal surface, $MJm^{-2}hr^{-1}$

$\bar{I}_d$ = monthly average hourly diffuse radiation received on a horizontal surface, $MJm^{-2}hr^{-1}$

$\bar{I}_\beta$ = monthly average hourly total radiation received on an inclined surface at β degrees to the horizontal, $MJm^{-2}hr^{-1}$

r_b = hourly ratio of the extraterrestrial radiation on a tilted surface to that on a horizontal surface for the month, dimensionless

$\bar{R}_b$ = daily ratio of the extraterrestrial radiation on a tilted surface to that on a horizontal surface for the month, dimensionless

r_T = ratio of the monthly average hourly to daily total radiation, dimensionless

r_d = ratio of the monthly average hourly to daily diffuse radiation, dimensionless

Greek Symbols

β = surface tilt from the ground, degrees

δ = declination, degrees

ϕ = latitude, degrees

ω = hour angle, degrees

ω_s = sunset hour angle for a horizontal surface, degrees

ω_s' = sunset hour angle for a tilted surface, degrees

ρ = albedo

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